applied mathematics practices
for the 21st century

FROM PERCENTAGES TO ALGEBRA
Using Authentic Problem Contexts

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Dear Teacher,

As mathematics teachers we are all challenged to provide our students with complex problems to solve. Our wish for their experience in mathematics is that they understand that real world scenarios involving mathematics often take more than one minute or one statement to solve. Where do we find those materials? Mathematics is used every day in business, industry, marketing, etc. to make decisions, plan ahead, and predict outcomes. Many textbook examples do not engage students or help them see the value of mathematics.

This book is our attempt to provide you with authentic, meaningful, engaging scenarios that showcase the importance of mathematics in the world. As state and national assessments (PSAT/SAT) are beginning to test more complex mathematics problems through the contexts of science and social studies, we find ourselves looking for practice to develop those critical thinking skills necessary for achievement.

According to the College Board website, the revised PSAT/SAT Mathematics Test "focuses in-depth on three essential areas of math: Problem Solving and Data Analysis, Heart of Algebra, and Passport to Advanced Math. Problem Solving and Data Analysis is about being quantitatively literate. It includes using ratios, percentages, and proportional reasoning to solve problems in science, social science, and career contexts. The Heart of Algebra focuses on the mastery of linear equations and systems, which helps students develop key powers of abstraction. Throughout the PSAT/SAT, you'll be asked questions grounded in the real world, directly related to work performed in college and career." This book focuses on using percentages to solve problems in business, industry, marketing, government, sports, and management. The Mathematics section on the PSAT/SAT "features multi-step applications to solve problems in science, social science, career scenarios, and other real-life situations. The test sets up a scenario and asks several questions that give you the opportunity to dig in and model it mathematically."

The scenarios in this book provide students with similar experiences. Students are given an authentic, meaningful scenario, asked to answer questions, dig deeper and model the situation mathematically. Some activities involve interpreting/analyzing data from a graph or table, while others involve creating a graph or table with given data or data they have collected themselves. Activities engage students in making transitions from percentages to algebraic expressions to solving equations.

This book provides a teacher guide for each of the activities. Each lesson begins with a brief Number Talk suggestion that directly relates to mindful mental mathematics for that particular lesson. The guide utilizes the lesson-planning framework of Thinking Through a Lesson Protocol which was developed through the collaborative efforts (lead by Margaret Smith, Victoria Bill, and Elizabeth Hughes) of the mathematics team at the Institute for Learning and faculty and students in the School of Education at the University of Pittsburgh. The lessons are structured in the Launch, Explore, Summary format. Suggestions have been provided in some of the columns. However, the "Who - Selecting/Sequencing" column (in the middle) is intentionally left blank for the teacher to make notes during the Explore portion of the lesson. It is intended for the teacher to sequence which groups/strategies will be presented and in what order during the Summary (most important) portion of the lesson. Much as a skillful conductor leads an orchestra, mathematics teachers must lead their students to a deeper understanding of the mathematics. This is done through collaborative discussions involving analyzing, critiquing, justifying, and making the mathematical connections. Each activity is expected to take between one and two class periods depending on your students' prior knowledge and the length of the class periods. Estimated time suggestions are provided, but can be easily adjusted to fit yours and your students' needs.

Enjoy!

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Introduction to Percentages

Percentages are the most common measure of performance in diverse settings. Percentages are so useful that the word is among the top 2,000 words used in the English language. Every child sees a percentage when he or she receives a grade on an exam. Every food label reports the percentage of each daily nutritional requirement. All high schools and colleges work to reduce their dropout percentage. Store sales are advertised as a percentage off of the price. In basketball the percentage of free throws made is a standard statistic. Every election poll reports the percentage of people who will vote for a specific candidate.

Percentages are linked to fractions and decimals. To determine the equal percent of a fraction, the fraction is simply multiplied by 100%. For example, one quarter is equal to 25%.

$$1/4 \times 100\% = 25\%$$

To determine the equal percent of a decimal, the decimal is multiplied by 100%. Thus, 0.25 is equal to 25%.

$$0.25 \times 100\% = 25\%$$

To find a percentage of a number, the number is multiplied by the equal decimal or fraction. For example, during flu season, 25% of the children might be absent on any day. In a class of 20 students, how many students would be absent? This number is found by

$$(1/4) \times 20 = 5$$

or

$$0.25 \times 20 = 5$$

Thus, 5 children would be absent in a class of 20 students.

It is useful to remember the relationship between some common fractions and percentages. These are summarized in Table 1. Since a dollar corresponds to 100 cents, there is a natural correspondence between percent and US coins. We believe that thinking about common US coins can help with this memorization. For example, ¼ corresponds to 0.25 and 25%. Similarly, the quarter coin represents one-fourth of a dollar. It is also called 25 cents. Three quarters is 75% of a dollar. In pennies it is 75 cents. Three-tenths is 0.30 or 30%. It corresponds to three dimes or 30 cents.

Some of the most confusing percentages and decimal equivalents involve values between 0 and 0.10. One one hundredth is 0.01. Students may forget the 0 that appears after the decimal place and before the one. In writing one percent, there is no need for a zero before the 1. Similarly, 0.05 corresponds to 5%.

<table>
<tr><th>Fraction</th><th>Decimal</th><th>Percent</th><td rowspan="11">The coin amount that corresponds to a percent or fraction of a dollar</td><th>Coins</th><th>Cents</th></tr>
<tr><td>1</td><td>1.00</td><td>100%</td><td>1 dollar</td><td>100</td></tr>
<tr><td>3/4</td><td>0.75</td><td>75%</td><td>Three quarters</td><td>75</td></tr>
<tr><td>½ or
2/4</td><td>0.50</td><td>50%</td><td>Half dollar
Two quarters</td><td>50</td></tr>
<tr><td>3/10</td><td>0.30</td><td>30%</td><td>Three dimes</td><td>30</td></tr>
<tr><td>1/4</td><td>0.25</td><td>25%</td><td>One quarter</td><td>25</td></tr>
<tr><td>1/5 or 2/10</td><td>0.20</td><td>20%</td><td>Two dimes</td><td>20</td></tr>
<tr><td>1/10</td><td>0.10</td><td>10%</td><td>One dime</td><td>10</td></tr>
<tr><td>1/20</td><td>0.05</td><td>5%</td><td>One nickel</td><td>5</td></tr>
<tr><td>1/100</td><td>0.01</td><td>1%</td><td>One penny</td><td>1</td></tr>
</table>

Table 1: Percentages and US coins

There are many common fractions that do not match US coins. For example, one-third is equal to the repeating decimal 0.333. There is no US coin equal to one-third of a dollar. The equivalent percent is 33.3%. When using percentages, analysts usually round to the nearest tenth of a percent or just the nearest percent. Similarly, one-seventh equals the repeating decimal with seven places, 0.142857142857... Its percent equivalent is 14.3%. One-eighth is simply 0.125 and does not need a repeating decimal representation. It is 12.5%. Table 2 matches common fractions with the nearest percentage.

		Percent	
Fraction	Decimal	Nearest 0.1%	Nearest %
1/3	0.3333…	33.3%	33%
1/6	0.1666…	16.7%	17%
1/7	0.142857…	14.3%	14%
1/8	0.125	12.5%	13%
1/9	0.1111…	11.1%	11%

Table 2: Common fractions and percentages that do not match coins

Summary of Examples: Percentages Everywhere

There are a total of 15 examples. They are designed to explore different ways of working with percentages. All of the examples are embedded in scenarios that involve decisions.

1. **Choose the Better Deal**: The first example starts with the most common textbook application of percentages, a price discount. The example also includes a fixed dollar discount. The student will compare the two and determine which is the better discount for different priced meals. This is facilitated with the introduction of an *algebraic expression* to be evaluated. The student then uses an *algebraic equation* to determine when the two discounts save the same amount of money.

2. **Making the Grade**: This example uses grades on tests that are reported as percentages, this allows for comparison of grades with a different number of questions. In this example, the student has received a score for his first exam. He wants to figure out was score he needs on the next exam to earn a specific letter grade. An algebraic equation is used to determine the minimum grade required to achieve his desired goal.

3. **Free Throw Percentages**: This example discusses how a competing team might use the free throw percentages for the Detroit Pistons as guide as to whom to foul late in the game. The major part of the example presents fictional data on free throws that the student uses to calculate the player's free throw percentage.

4. **Dropping Out – When 0% is best:** In the previous examples a higher percentage was better. In this example the focus is on the percentage of students who drop out before completing high school. A smaller percentage is better. Students are asked to calculate percentages to determine which dropout prevention program is better.

5. **Special Ops:** In this example, two officers discuss the challenges of volunteers passing two rigorous weeks of different types of training for a special mission. The student will evaluate whether it is better to have the training with the higher failure rate first or last. This includes an economic analysis of the order of training. Students will use algebra to

determine how many volunteers are needed to enter training in order to meet the need for 36 soldiers who have passed both weeks of training.

6. **Growing a business – Compound Percentages**: Growing a business is one of the few applications in which percentages can exceed 100%. In this example students will evaluate two different marketing programs. One program increases the number of customers by a percentage. Multiple weeks of growth illustrate the concept of compound percentages. The other marketing program adds a fixed number of customers each week. Algebra is used to estimate the number of customers several weeks later as the two programs are compared.

7. **McFadden Restaurant – Changing Revenue:** This example discusses the monthly sales of a restaurant. The example addresses the misconception that if sales increase by 20% one month and then decrease by 20% the next month, the final number is the same as the starting number. It is not. This example also uses algebra to determine total revenue used to calculate the franchise fee.

8. **Managing by Percentages**: This example is actually a collection of several small examples. In each case the decision maker is allocating a resource based on the percentage demand for different products. In several examples, the resource is space in the store. In another example, it is advertising dollars. This collection introduces the student to a commonly used demographic factor, the percentage of women and men shoppers. It also discusses the mix of customers who prefer organic fruits and vegetables and are willing to pay more organic foods.

9. **Multiple Flavor Ice Cream Sales:** In previous examples there were only two possibilities. In this example the manager is deciding how much of each flavor of ice cream to stock. There are four flavors to choose from each with a different percentage of demand. This example involves an economic analysis of the alternative plans for stocking flavors. Students will need to work with numbers that do not always produce whole numbers of a product. Algebra is introduced to determine a break-even point for ordering a whole liter of ice cream even if all of it cannot be sold.

10. **Ordering Hoodies – Multiple Percentages:** This example continues the development of contexts with more than two percentages. The primary decision is how many items of each size should be ordered. The student is introduced to the concept of a pie chart that is used to show and compare percentages of sizes for men and women. One complication is that the calculations often result in non-whole numbers. However, only a whole number of items can be ordered. The problem also involves working with pairs of percentages, sizes and colors.

11. **Compound Growth of APP Users:** In this example, two high schoolers develop a successful game app. The number of users grows by 25% each month. The example involves compounding percentages to determine the number of users several months from now. The example involves extensive economic analysis. The developers are selling advertising in order to raise money to pay for living on campus rather than commuting. This example includes a number of graphs that the student is asked to read and interpret.

12. **Compound Percentage Decline of Homeless Veterans:** Homelessness is a national problem that is being worked on by both federal and state agencies. This example presents two alternatives for reducing homelessness among veterans. One program reduces homelessness by 17% a year. The other helps a fixed number of veterans each year. The example explores the relative effectiveness of the two programs over a multi-year time period. Graphs and algebraic models are an important aspect of this example.

13. **Weighted Averages and Grades:** This is an extension of the earlier example involving grades. In this example the various components of the grades are not equally weighted. In addition, there will contexts with three elements to calculating the grade. An algebraic equation is used to determine the minimum percentage needed on the final element to bring the overall grade above a specific threshold.

14. **Open24 Slushy Sales – Weight Values by Percentage:** This example presents data on the demand for different sized iced drinks. Calculating overall performance of the store involves taking a weighted sum of multiple sized drinks. The store manager is trying to decide between two different ad campaigns that can impact sales of these iced drinks.

15. **Congressional Districts** – Combining Percentages: This example explores how the design of a congressional district could affect the likelihood of a Republican or Democrat winning the congressional election. The region is made up of 12 geographic units with different percentages of Republican and Democratic voters. The state must group these 12 units into four congressional districts. The student will evaluate two different designs for the four districts. The student is then challenged to create a different plan that is more fair to both sides.

All 15 examples use percentages to make decisions in a meaningful context. In designing these examples, we strove to include the use of other important math skills in a natural way. Every example presents data in a table format. A number of the latter examples also include line graphs and pie charts. Seven of the examples introduce algebraic expressions and equations to determine a specific value of interest. At the end of all of the examples, we present a simple project idea for collecting data related to the context of the example.

0	Introduction	Concepts	Table	Algebra	Graph Chart	Project Idea
1	Better deal	Compare percent and fixed value of coupons	Y	Solve Equation	N	Y
2	Making the grade	Best is 100% - weighted scores	Y	Solve Equation	N	Y
3	Free throw percentages	Best is only 90%	Y	N	N	Y
4	Dropping out of high school	Best is 0 %	Y	N	N	Y
5	Special operations	Order of percent change no impact	Y	Solve Equation	N	Y
6	Growing lawn service	Compare percent and fixed value	Y	Solve Equation	N	Y
7	McFadden restaurant	Fallacy of equal + and – percentage change	Y	N	Y	Y
8	Managing by percentage	More than two percentages and non-whole number answers	Y	N	N	Y
9	Ice cream	More than two percentages and non-whole number answers	Y	N	Y	Y
10	Ordering Hoodies	Multiply two percentages and non-whole number answers	Y	N	Y	Y
11	App growth	Compound percentages and financial analysis	Y	Set up Formula	Y	Y
12	Homeless veterans	Compound percent vs fixed	Y	Set up Formula	Y	Y
13	Grades	Weighted Scores - 3 values	Y	Solve Equation	N	Y
14	Different sized Slushies	Weighted sum of percentages	Y	N	Y	Y
15	Congressional districts	Combine geographies and their percentages	Y	N	Y	Y

Activity 1: Choosing the better deal

Mathematical Goals

The student will use percentages to determine which coupon is better.

In part I the student will:

- Read an advertisement and interpret the information
- Perform operations with percentages
- Complete a table
- Work with percentages in a meaningful context familiar to students

In part II, appropriate for students who have been introduced to basic algebra, the student will:

- Transition to the use of Algebra to answer a question
- Evaluate an algebraic expression for different values of the variable
- Solve an algebraic equation

Before the lesson (5-10 minutes)

Number talk possibilities:

Select two or three depending on student abilities.

- Find 10% of $30.00.
- Find 10% of $23.00.
- Find 10% of $23.50.
- Find 5% of $30.00.
- Find 5% of $23.00.
- Find 5% of $23.50.

Choosing the better deal

Jimmy’s Coney Island sends out the coupon below in a weekly mailer.

Chris went to Coney for a quick lunch. His bill for the meal was $5.00. He handed the cashier the 15% off coupon.

1. How much money did he save? How much did he pay for lunch?

His friend Clarissa wondered why Chris had not used the $2 coupon instead. Chris told her to look carefully at the rule for using the $2 coupon.

2. Why couldn’t Chris use the $2 coupon?

3. If Chris could have used the $2 coupon what percent of the bill would he have saved?

4. Why do you think Coney Island limited the use of the $2 coupon?

5. Ramon, Jennifer and Marianne were having a hearty breakfast at Jimmy's and the total bill came to \$35. Which coupon should they use to save the most money? Justify your answer by finding the savings for both coupons and then comparing them.

6. Donald was having breakfast alone and his total bill was \$10. Which coupon should he use to save the most money? Justify your answer by finding the savings for both coupons and then comparing them.

Chris' friend Clarissa came up with a different way to think about the problem. She wanted to organize her thinking. She let y represent the total bill for any meal. The letter y is called a *variable*, because its value can change. Different meals can have different total bills. To find 15% of a number, you can multiply the number by 0.15. She multiplies y by 0.15 to represent the amount taken off the total bill if they use the 15% off coupon:

$0.15y$ = the amount taken off the total bill if you use the 15% off coupon.

If a dinner costs \$30, the variable, y, is replaced by \$30 in the expression. Then the amount off is

0.15(\$30) = \$4.50, and the dinner costs \$30.00–\$4.50 = \$25.50

If a dinner costs \$18, then y is replaced in the expression by \$18. The expression representing the amount off is 0.15(\$18) = \$2.70, and the dinner costs \$18.00–\$2.70 = \$15.30.

7. Fill in Table 1 for y = \$11.00 and \$15.00.

Total Bill y	**15% off Coupon**		**$2.00 off Coupon**	
	Amount Off $0.15y$	**Final Bill $y - 0.15y$**	**Amount Off $2.00**	**Final Bill $y - 2.00$**
$5.00	(0.15)$5.00=$0.75	$5.00-$0.75=$4.25	Not Applicable	Not Applicable
$10.00	(0.15)$10.00=$1.50	$10.00–$1.50=$8.50	$2.00	$10.00–$2.00=$8.00
$11.00				
$15.00				
$18.00	(0.15)$18.00=$2.70	$18.00–$2.70=$15.30	$2.00	$18.00–$2.00=$16.00
$30.00	(0.15)$30.00=$4.50	$30.00–$4.50=$25.50	$2.00	$30.00–$2.00=$28.00
$35.00	(0.15)$35.00=$5.25	$35.00–$5.25=$29.75	$2.00	$35.00–$2.00=$33.00

Table 1: Comparison of two coupons for different priced meals

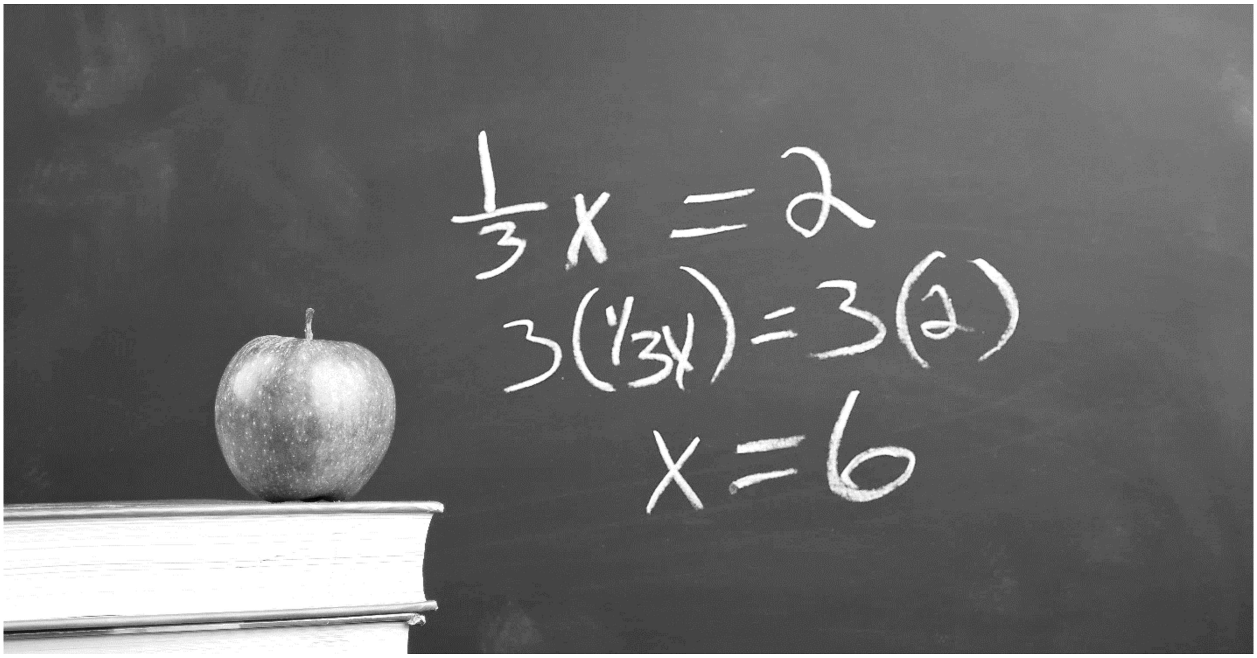
8. Try some different values for y in the table above to find the prices when the $2.00 off coupon is the better deal and when the 15% off coupon is the better deal. Is there any price when the two coupons result in the same deal? If so, what is that price?

Use the data in Table 1 to fill in the following two statements:

9. When the bill is less than _______ the better coupon is ______ off.

10. When the bill is more than _______ the better coupon is ______ off.

Part II – Algebra – Better than trial and error



By substituting numbers for the variable in the table, it is possible to see a pattern when one coupon is better than the other. However, this method does not easily find the exact value at which the two coupons are worth the same. In the above discussion, the representation $(0.15)y$ is called an algebraic expression. This algebraic expression is useful, because it represents the amount taken off any meal when the 15% off coupon is used. It also can be used to answer the question, "When is the amount taken off the same for both coupons?" To answer that question, we can write an *algebraic equation*. We set the 15% off algebraic expression equal to \$2, the constant amount taken off when using the other coupon.

$0.15y = \$2$

algebraic expression: a mathematical expression that consists of variables, constant numbers and mathematical operations. (Addition and multiplication are examples of operations.) The value of an algebraic expression changes as the value of the variable changes.

We now need to find the value of y that makes the algebraic expression equal to \$2. Finding the correct value of y is what solving an algebraic equation means. To do this, we apply the same mathematical operations to both sides of the equation until we are left with just $1y$ on the left hand side. In the equation, y has been multiplied by 0.15. To undo multiplying by 0.15, we use the inverse operation, division. When we divide both sides of the equation by 0.15, the left hand side becomes 1y, because 0.15y/0.15 = 1y. The two sides of the equation are still equal, so

$1y = \$2 \div 0.15 \approx \$13.33$.

11. To show that this answer is correct calculate (0.15)(\$13.33). Is the answer \$2?

12. Which coupon would be better if the meal cost

    a. \$13.34?
    b. \$13.30?
    c. \$13.36?
    d. \$13.29?
    e. \$13.37?

<u>Solving an algebraic equation</u>. To solve an algebraic equation, change the equation so that the variable $1y$ is alone on one side of the equation, and the other side is a number. To do this, carry out the same mathematical operations (+, –, x, ÷) in the same order on both sides of the equation. Anytime you carry out the same operation on both sides of the equation, the quantities will still be equal. We can subtract or add the same number to each side of an equation, and the two sides will still be equal. We can multiply or divide both sides of an equation by the same number, and the two sides will still be equal. This process of performing the same operations on both sides of an equation to reach the goal of just 1y on one side and a number on the other is one of part of learning algebra. When you learn the process, you have the ability to solve an equation. You will develop this skill as you study different kinds of algebraic equations. You will learn to recognize patterns to decide what set of operations should be applied to both sides of the equation.

13. Based on your previous answers, for which prices does
    a. the $2.00 off coupon give the most off?
    b. the 15% off coupon give the most off?
    c. the two coupons give the same amount off?

Jimmy's Coney Island determined the coupons they were offering reduced their profits too much. They decided to offer coupons of 10% off or $1.50 off.

14. For what priced meals should you use the 10% coupon? When should you use the $1.50 coupon? When are the savings for both coupons equal?

Project Idea:

Look for an advertisement that has a restriction on using the discount. What is the restriction? Does it make sense?

Look for advertisements that offer both a percent discount and a fixed dollar discount. Determine the range of prices for which the percent coupon is better than the fixed amount.

Practice problems

1. You are purchasing a gift online and the total bill is $24.52. The site offers a discount coupon of 20% off the total bill or free shipping and handling for orders over $20. The shipping and handling fee for this order is quoted at $5.29. Which offer saves you the most?

2. The online site above offers gift wrapping for $2.00. You decide to have your gift wrapped which brings the total bill to $26.52. Which offer saves you the most?

3. Rosie's Cafe offers a percentage discount on your birthday equal to your age. You must show valid ID with birthdate to qualify. You plan to eat at Rosie's on your 18th birthday. You also found a $4 off coupon for meals at Rosie's over $16.00. Your total bill is $17.65. Which discount saves you the most?

4. On your 18th birthday at Rosie's, what price for the total bill is the discount with the $4 off coupon and the 18% birthday discount the same?

5. A car dealership offers two choices as discounts. The first choice is $2,016 dollars off the price for 2016 model year cars. The second choice is an 11% discount on your vehicle of choice. You have decided to purchase a slightly used Kia from 2016. The purchase price is $18,500. Which discount offer saves you more money?

6. You are looking into buying a new car. You have a family and friends discount from Ford Motor Company and that will save you 17% off the sticker price. Ford is presently running a sale to reduce inventory. The sale involves a "cash discount" of $4,000. The vehicle you have in mind has a sticker price of $22,994. Which discount will save you more?

Activity 1:
Teachers' guide

Thinking through a lesson protocol

Standards:

6.RP.A.3.C: Find a percent of a quantity as a rate per 100 (e.g., 30% of a quantity means 30/100 times the quantity); solve problems involving finding the whole, given a part and the percent.

6.EE.A.2.A: Write expressions that record operations with numbers and with letters standing for numbers. *For example, express the calculation "Subtract y from 5" as 5 - y.*

6.EE.A.2.C: Evaluate expressions at specific values of their variables. Include expressions that arise from formulas used in real-world problems. Perform arithmetic operations, including those involving whole-number exponents, in the conventional order when there are no parentheses to specify a particular order (Order of Operations).

6.EE.A.6: Use variables to represent numbers and write expressions when solving a real-world or mathematical problem; understand that a variable can represent an unknown number, or, depending on the purpose at hand, any number in a specified set.

6.EE.A.7: Solve real-world and mathematical problems by writing and solving equations of the form $x + p = q$ and $px = q$ for cases in which p, q and x are all nonnegative rational numbers.

Mathematical Practices:

MP1: Make sense of problems and persevere in solving them.
MP2: Reason abstractly and quantitatively.
MP3: Construct viable arguments and critique the reasoning of others.
MP6: Attend to precision.

Setting up the problem - Launch	
Selecting tasks/goal setting	(15-20 minutes) Briefly discuss or ask students in a whole group setting for ideas when percentages are used in real life. Prior to working this scenario, students should read the "Introduction to Percentages."
Questions	Can you think of a time that percentages are used? Think on your own, what is the relationship between 25% and a quarter? Share it with a friend. Students could build a table similar to the one given in the "Introduction to Percentages."

<table>
<tr><th colspan="3">Monitoring student work - Explore</th></tr>
<tr><th>Strategies and misconceptions- Anticipating</th><th>Who - Selecting and sequencing</th><th>Questions and statements - Monitoring</th></tr>
<tr><td>(20 minutes) Have students look at coupon from Jimmy's Coney Island and answer questions #1-6. Share results with whole group.</td><td></td><td>Explain the differences and the similarities between the coupons in the flyer. Note: Be sure to mention that the $2.00 off coupon may only be used when the total bill is $10 or more.</td></tr>
<tr><td>(20 minutes) Read text section after question #6 and before question #7 as a whole class. Then solve and discuss questions #6-10.</td><td></td><td></td></tr>
<tr><td rowspan="2">(20 minutes) Read text section in Part II as a whole class. Then solve questions #11-14.</td><td rowspan="2"></td><td>Depending on students' prior knowledge of algebraic expressions the time spent on this section may vary. Spend more time if this is their first exposure, less if they are comfortable with expressions.</td></tr>
<tr><td>When solving equations, stress the importance of inverse operations (undoing) and performing the same operation on both sides of the equation sign to keep the quantities equal.</td></tr>
</table>

Monitoring individual student work - Explore		
Strategies and misconceptions- Anticipating	**Who - Selecting and sequencing**	**Questions and statements - Monitoring**
For off-task students or for students that seem to be self-conscious about you listening to them share.		I am just listening or looking to find out how you are working on the problem. This helps me think about what we will do later. What do you think is the Big Idea in the Introduction to Percentages reading?
For students that appear to be stuck.		Can you tell me a little about your reading? How could you describe the relationship between percents and coins/dollars? How would you describe the problem in your own words? What facts do you have? Could you try it with simpler numbers? Fewer numbers?
For students that want to ask you questions, these are ways to uncover their thinking and judge to what extent you want to respond.		Tell me what you've thought about so far. What do you know? Why are you interested in more information about that? Let me say a little about that part.

Managing the discussion – Summarize	
Parts of discussion - Connecting	**Questions and statements - Connecting**
Launching the discussion: Select the problems in questions #7-10 that students are struggling with or you wish to share out.	Will team 1 start us off by sharing one way of working on this problem? Please raise your hand when you are ready to share your solution. What did you do first when you were working on this problem? Let’s start by clearing up a few things about the problem. Let’s list some key parts in this problem? What was unclear in the problem?
Eliciting and uncovering student strategies	Joe would you be willing to start us off? What have you found so far? Can you repeat that? Can you explain how you got that answer? How do you know? Walk us through your steps. Where did you begin? Can you show us?
Focusing on mathematical ideas	Can you explain why this is true? Does this method always work? How is Bob’s method similar to Kelly’s method? What do all the solutions have in common? What would happen if I changed the numbers to _____?
Encouraging interactions	Do you agree or disagree with Kahlil’s idea? What do others think? Would someone be willing to repeat what Tom just said? Would anyone be willing to add on to what Sue just said?
Concluding the discussion	Can anyone tell me some of the big ideas that we learned today? How would you explain what we learned today to a 5^{th} grader? Some of the key points from our discussion today are . . . Tomorrow we will continue our exploration of ______ beginning with the idea from today that ___________.
Post lesson notes	You may wish to assign the practice problems that you feel would benefit the students.

Answers – choosing the better deal

1. How much money did he save? How much did he pay for lunch?

   Saved 0.15×$5 = $0.75. Total cost = $5.00 - $0.75 = $4.25

2. Why couldn't Chris use the $2 coupon?

   The $2 coupon can only be used for purchases of $10 or more.

3. If Chris could have used the $2 coupon what percent of bill would he have saved?

   ($2/$5) (100%) = 40% savings

4. Why do you think Coney Island limited the use of the $2 coupon?

   The restaurant would make no money on the meal when the percent discount is large compared to the bill. Only when the bill is $10 or more do they feel there is enough profit to afford a $2 discount.

5. Ramon, Jennifer and Marianne were having a hearty breakfast at Jimmy's and the total bill came to $35. Which coupon should they use to save the most money? Justify your answer by finding the savings for both coupons and then comparing them.

   Solution 1

   15% OFF Coupon: *$2.00 OFF Coupon:*

   You will save 15% of $35.00? *You will save $2.00*

   *0.15 * $35.00 = $5.25*

   You save more with the 15% OFF Coupon

   Solution 2

   15% OFF Coupon: *$2.00 OFF Coupon:*

   You will pay 85% of the bill. *You will get $2.00 off your bill.*

   0.85 × $35.00 = $29.75 *$35.00 – $2.00 = $33.00*

   The 15% OFF Coupon results in a smaller bill and saves the most money.

6. Donald was having breakfast alone and his total bill was $10. Which coupon should he use to save the most money? Justify your answer by finding the savings for both coupons and then comparing them.

   Solution 1

   15% OFF Coupon: *$2.00 OFF Coupon:*

   You will save 15% of $10.00? *You will save $2.00*

   0.15 × $10.00 = $1.50

   The $2.00 OFF Coupon results in a smaller bill and saves the most money.

Solution 2

15% OFF Coupon: *\$2.00 OFF Coupon:*

You will pay 85% of the bill. *You will get \$2.00 off your bill.*

0.85 × \$10.00 = \$8.50 *\$10.00 - \$2.00 = \$8.00*

The \$2.00 OFF Coupon saves you the most money.

7. Fill in the Table 1 below for *y* = \$11.00 and \$15.00.

Total Bill y	***15% off Coupon***		***\$2.00 off Coupon***	
	Amount Off 0.15y	***Final Bill y – 0.15y***	***Amount Off \$2.00***	***Final Bill y – 2.00***
\$5.00	*0.15(\$5.00)=\$0.75*	*\$5.00–\$0.75 =\$4.25*	*Not Applicable*	*Not Applicable*
\$10.00	*0.15(\$10.00)=\$1.50*	*\$10.00–\$1.50 =\$8.50*	*\$2.00*	*\$10.00–\$2.00=\$8.00*
\$11.00	*0.15(\$11.00)=\$1.65*	*\$11.00–\$1.65 =\$9.35*	*\$2.00*	*\$11.00–\$2.00=\$9.00*
\$12.00	*0.15(\$12.00)=\$1.80*	*\$12.00-\$1.80 =\$10.20*	*\$2.00*	*\$12.00–\$2.00 =\$10.00*
\$13.00	*0.15(\$13.00)=\$1.95*	*\$13.00–\$1.95 =\$11.05*	*\$2.00*	*\$13.00–\$2.00 =\$11.00*
\$15.00	*0.15(\$15.00=\$2.25*	*\$15.00–\$2.25 =\$12.75*	*\$2.00*	*\$15.00–\$2.00 =\$13.00*
\$18.00	*0.15(\$18.00)=\$2.70*	*\$18.00–\$2.70 =\$15.30*	*\$2.00*	*\$18.00–\$2.00 =\$16.00*
\$30.00	*0.15(\$30.00)=\$4.50*	*\$30.00–\$4.50 =\$25.50*	*\$2.00*	*\$30.00–\$2.00 =\$28.00*
\$35.00	*0.15(\$35.00)=\$5.25*	*\$35.00–\$5.25 =\$29.75*	*\$2.00*	*\$35.00–\$2.00 =\$33.00*

Table 1: Comparison of two coupons for different priced meals

8. Try some different values for *y* in the table above to find the prices when the \$2.00 off coupon is the better deal and when the 15% off coupon is the better deal. Is there any price when the two coupons result in the same deal? If so, what is that price?

Answers will vary. See the calculations added to the Table 1.

9. When the bill is less than _______ the better coupon is ______ off.

   When the bill is less than \$10 the better/only coupon is 15% off. For bills between \$10 and approximately \$13.29, the better coupon is the \$2.00 off coupon.

10. When the bill is more than _______ the better coupon is ______ off.

    When the bill is more than \$13.37, the better coupon is the 15% off.

Part II Algebra

11. To show that this answer is correct calculate (0.15)(\$13.33). Is the answer \$2?

    (0.15)(\$13.33) = \$1.9995 ≈ \$2.00; yes, when rounded-off to the nearest penny, which is the smallest coin denomination.

12. Which coupon would be better if the meal cost is

    a. \$13.34? *(0.15)(\$13.34) = \$2.001 ≈ \$2.00; the coupons are equal in value*

    b. \$13.30? *(0.15)(\$13.30) = \$1.995 ≈ \$2.00; the coupons are equal in value*

    c. \$13.36? *(0.15)(\$13.36) = \$2.004 ≈ \$2.00; the coupons are equal in value*

    d. \$13.29? *(0.15)(\$13.29) = \$1.9935 ≈ \$1.99; the \$2.00 off coupon is better*

    e. \$13.37? *(0.15)(\$13.37) = \$2.0055 ≈ \$2.01; the 15% off coupon is better*

13. Based on your answers to questions 5 and 6, for which prices does
    a. the \$2.00 off coupon give the most off? *Prices ≤ \$13.29*
    b. the 15% off coupon give the most off? *Prices ≥ \$13.37*
    c. the two coupons give the same amount off?
       Because of rounding to the nearest penny, the discount is the same for prices from \$13.30 to \$13.36

14. For what priced meals should you use the 10% coupon? When should you use the \$1.50 coupon? When are the savings for both coupons equal?

    To answer that question, we can write an algebraic equation. We set the 10% off algebraic expression equal to \$1.50, the constant amount taken off when using the other coupon.
    0.1y = \$1.50
    We divide both sides by 0.1.
    0.1y / 0.1= \$1.50/0.1
    y = \$15.00
    However, because of rounding, values close to \$15.00 will also yield a \$1.50 discount.

Solutions to practice problems

1. You are purchasing a gift online and the total bill is $24.52. The site offers a discount coupon of 20% off the total bill or free shipping and handling for orders over $20. The shipping and handling fee for this order is quoted at $5.29. Which offer saves you the most?

   20% of $24.52 is $4.90. That discount is less than $5.29, therefore, it would be better to take the discount for the shipping and handling of $5.29.

2. The online site above offers gift wrapping for $2.00. You decide to have your gift wrapped bringing the total bill to $26.52. Which offer saves you the most?

   20% of $26.52 is $5.30. That is one penny more than the shipping and handling discount of $5.29, therefore, to save the one penny, you would take the 20% off discount.

3. Rosie's Cafe offers a percentage discount on your birthday equal to your age. You must show valid ID with birthdate to qualify. You plan to eat at Rosie's on your 18th birthday. You also found a $4 off coupon for meals at Rosie's over $16.00. Your total bill is $17.65. Which discount saves you the most?

   The discount for the $17.65 bill would be 18% of $17.65 which is $3.177. The $4.00 off coupon would be more of a savings.

4. On your 18th birthday at Rosie's, what price for the total bill is the discount with the $4 off coupon and the 18% birthday discount the same?

   18% of "n" is $4.00 then 4.00/.18 = $22.22. In order for the 18% discount to match the $4.00 discount, the bill would have to be $22.22.

5. A car dealership offers two choices as discounts. The first choice is $2,016 dollars off the price for 2016 model year cars. The second choice is an 11% discount on your vehicle of choice. You have decided to purchase a slightly used Kia from 2016. The purchase price is $18,500. Which discount offer saves you more money?

   11% of $18,500 is $2,035. Comparing that to the $2,016 cash giveaway, the $2,035 saves you more, precisely $19 more.

6. You are looking into buying a new car. You have a family and friends discount for Ford Motor Company and that will save you 17% off the sticker price. Ford is presently running a sale to reduce inventory. The sale involves a "cash discount" of $4,000. The vehicle you have in mind has a sticker price of $22,994. Which discount will save you more?

   17% of $22,994 is $3,908.98. That is less than the $4,000 discount, so that would save you more, exactly $91.02 more.