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A mathematics course with *REAL* answers!

Volume 1: Algebraic Modeling

3	Profit Maximization Problem	Plan Skateboard Production				
4						
5	Decision Variable	Sporty (x1)	Fancy (x2)	Pool-Runner (x3)		
6	Decision Values [weekly production rate]	0	104	210		
7					Total Profit	
8	Objective Function [Profit ($)]	15	35	20	7840	
9						
10	**Constraints**					
11	Shaping Time (minutes)	5	15	4	2400	<=
12	Truck Availability	2	2	2	628	<=
13	North American Maple Veneers	0	7	0	728	<=
14	Chinese Maple Veneers	7	0	7	1470	<=
15						

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Maximize Profitability

Buy a Used Car

Standing in Line

Choose a College

Form Teams

Maximize Political Ads

Select a Cell Phone Plan

Minimize Pollution

Thomas Edwards, Ph.D. | Kenneth Chelst, Ph.D.

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Dr. Thomas Edwards

Thomas G. Edwards is the Associate Dean for Research in the College of Education at Wayne State University. He completed his Ph.D. in mathematics education at Ohio State University in 1994. Prior to his Ph.D., Edwards taught for more than two decades as a high school mathematics teacher.

Contact him at:
t.g.edwards@wayne.edu

Dr. Kenneth Chelst

Kenneth R. Chelst is a professor of operations research in the Department of Industrial and Systems Engineering at Wayne State University in Detroit, MI. He completed his Ph.D. in operations research at MIT in 1975. Dr. Chelst has consulted on more than 100 projects in public safety services and engineering management.

Contact him at:
kchelst@wayne.edu

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INFORMS (The Institute for Operations Research and the Management Sciences) provides funding for teachers workshops across the country. These one-day overview and three-day intensive workshops provide an introduction and basic education about the material presented in "**When Will I Ever Use This?**"

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Wayne State University

Dr. Thomas Edwards
Dr. Kenneth Chelst
Mr. Jay Johnson

North Carolina State University

Dr. Karen Keene
Dr. Karen Norwood
Dr. Robert Young
Dr. Molly Purser
Dr. Amy Craig-Reamer

University of North Carolina Charlotte

Dr. David Royster
Dr. David Pugalee
Ms. Sarah Johnson

Researchers

Dr. Saman Alaniazar
Dr. Hatice Ucar
Dr. Richelle Dietz
Dr. Krista Holstein
Mr. Will Hall
Ms. Zeyneb Yurtseven

Teachers

Derek Blackwelder, Jamie Blanchard James Bruckman, Kevin Busfield, Robert Butler, Molly Charles, Suzanne Christopher, Kristina Clayton, Krystle Corbin, Dean DiBasio (teacher writer), Robin Dixon, Caroline Geel, Michael Gumpp, Breanna Harrill, Jack Hunter, Amy Johnson, Jennifer Johnson, Jenise Jones (teacher writer), Anna Kamphaus, Chris Kennedy, George Lancaster (teacher writer), Craig Lazarski, Jeffrey Loewen, Charles Ludwick, Margaret Lumsden, Leon Martin, Cora McMillan, Kristen Meck (teacher writer), Karen Mullins, Erica Nelson, Bethany Peters, Joseph Price, Angela Principato (teacher writer), John Pritchett, Tyler Pulis, Paul Rallo, Tonya Raye, Kym Roberts, William Ruff, Amy Setzer, Rob Sharpe, Christy Simpson, Catherine Sitek, Cheryl Smith, Julia Smith, Wendy Carol Srinivasan, Susan Sutton, Greg Tucker, Laura Turner, Thad Wilhelm (teacher writer), Tracey Walden, Tracey Weigold, Lanette Wood

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When Will I Ever Use This?

Volume I: Algebraic Modeling

Thomas Edwards and Kenneth Chelst

Table of Contents

Chapter 1: Make Hard Decisions—Multi-Criteria Decision Making (MCDM)

We all face decisions in our jobs, in our communities, and in our personal lives. For example,

- Where should a new airport, manufacturing plant, power plant, or health care clinic be located?
- Which college should I attend, or which job should I accept?
- Which car, house, computer, stereo, or health insurance plan should I buy?
- Which supplier or building contractor should I hire?

Multi-criteria decision making (MCDM) is used when one needs to make a hard decision with many criteria. The method introduced in this chapter is a structured methodology designed to handle the tradeoffs among multiple criteria. MCDM is a systematic approach to quantify an individual's preferences. Measures of interest are rescaled to numerical values on a 0–1 scale, with 0 representing the worst value of the measure and 1 representing the best. The decision maker assigns weights to each criterion to reflect the relative value of each criterion to the decision maker. This allows the direct comparison of many diverse measures. In other words, with the right tool, it really is possible to compare apples to oranges! The result of this analysis is a rank ordering of the alternatives that reflects the decision makers' preferences.

This chapter uses basic mathematics skills in an intellectually challenging environment. It was designed to provide all students with an appreciation that they can successfully apply their math skills to sophisticated decisions.

Chapter 2: Optimize Product Mix with Linear Programming (Maximization)

Chapter 2 is the introductory chapter for the optimization topics in chapters 2-7. While the succeeding chapters will extend this topic to minimization, integer programming, and binary programming, the skills in this maximization of linear programming (LP) chapter are the basic tools that will be used throughout. There is an introductory activity that involves using Lego to model the decision of making chairs and tables. This provides the students with a concrete understanding of decision variables and constraints. The problem context for this first substantive problem involves assembling two types of computers with different profit margins and labor requirements. Students are led through a graphical solution to a two decision variable problem involving two constraints. This is called the corner point principle. Lastly, the problem is expanded to include more decision variables, x_3, and x_4, to represent two additional configurations of computers. Once there are more than two decision variables, the problem cannot be solved graphically. In the next section of the text, students learn how to use SOLVER, a standard add-in to EXCEL, to solve mathematical programming problems. From this point on, Excel is a critical element of each chapter.

Chapter 3: Analyze Optimal Solutions—Sensitivity Analysis
This chapter revisits the three examples from chapter 2. In addition to solving problems, analysts are often interested in learning how sensitive their solutions are to changes in the parameters of the problem. Consider the computer assembly problem in the chapter 2. How sensitive is the solution to changes in the amount of profit that is made on each type of computer? What would be the effect of increasing the amount of available installation time or testing time? Questions such as these are part of what is called **sensitivity analysis**. This chapter is designed around the sensitivity analysis report that is part of Solver. Students learn how to explore and answer diverse what if questions.

Chapter 4: Minimize Calories or Cost with Linear Programming
This chapter presents three decision contexts in which the goal is to minimize the objective function. The first example was taken from a UN report and involves designing a nutritious diet for children in Malawi. The number of constraints and wide range of units of measures adds to the complexity of the problem. The second decision context motivates the need to use double subscripted variables to represent a pollution control decision in the Wisconsin watershed. The final decision involves more complex constraints that require students to manipulate equations to convert them to a standard form required for the Excel implementation of the model.

Chapter 5: Optimize Effectiveness or Cost with Integer Programming
In the previous chapters on linear programming, the decision variables were things that can be measured continuously such as production rates, grams of a food source, or 100-gallons of gasoline. However, in some contexts, the decision variable could be restricted. For example, a manager might need to know how many of each type of worker to hire. In this chapter, we will discuss how to solve mathematical programming problems in which the decision variables must be restricted to integer values. We will also discuss why such a restriction makes a difference in how the problem is solved and how its solution is analyzed. The chapter begins by investigating integer and non-integer solutions to linear equations in the context of purchasing advertisements to support a political campaign. The second example involves scheduling workers and supervisors at a fast food restaurant. The final decision context involves planning the shipment of truckloads of oranges to different markets in the Midwest. It illustrates the basic concepts of logistics planning. This chapter reinforces the core algebraic modeling skills of defining decision variables, framing the objective function and structuring the constraints.

Chapter 6: Optimize Selection with Binary Programming
Binary Integer Programming (BIP) problems have the same basic features as other mathematical programming problems: a set of decision variables, an objective function, a system of constraints. The distinguishing feature of BIP problems is that the possible values of the decision variables are limited to zero and one. These are called *binary decision variables*. Chapter 6 opens with a simple investment with just two decision variables and a single constraint. Students are asked to consider different values for the RHS of the constraint and investigate the implications of those changes on the optimal solution of the BIP problem. The first substantive example involves flipping houses. This example uses a complex spreadsheet to capture all of the information required to select which houses to buy and fix up.

Assignment problems are a special case of binary integer programming. Assignment problems involve matching a number of agents (e.g., athletes, students, machines) to a number of tasks (e.g., events, teams, jobs). This information is represented using matrices. If there are *m* agents and *n* tasks, then an $m \times n$ matrix of binary decision variables assigns agents to tasks. Assignment problems have a second $m \times n$ matrix, called a *cost matrix* that shows the "cost" (e.g., time, distance, monetary cost) associated with each agent performing each task.

Chapter 7: Find Optimal Locations with Algorithms
Amy Craig, Ph.D. - UNC Wilmington

This chapter focuses on different types of location problems. Location decisions arise in many contexts. All fast food companies, oil companies, drugstore chains or other retail outlets routinely evaluate locations for new facilities. Similar decisions are made in the public sector with regard to the location of libraries, fire stations, school buildings, and health care clinics. In some instances a simple measure of travel distance suffices to guide the decision. In other instances multiple criterion are used as in chapter 1 of this text.

The chapter starts with a simplified example involving two smoothie stands located along a single stretch of road. This example introduces the concept of minimizing the average distance traveled. Next we explore where to locate a small warehouse to store excess inventory for a downtown store. The third decision involves the location of a warehouse along a major interstate. Trucks from this warehouse are to make deliveries to different cities along the interstate. We then move to two dimensions as the decision involves locating a food stand in a downtown area. In some contexts, the preferred measure involves ensuring that all potential users of a service are within a fixed distance of the nearest facility. Any set of users that are within the prescribed distance are said to be ***covered*** by that facility. The final example utilizes binary integer programming to find the optimal locations for disaster response facilities to provide coverage of the region.

Chapter 8: Waiting in Line with Polynomials – Non-linear functions and Queues

There are relatively few broadly relevant applications of non-linear functions in high school mathematics curriculum. Mathematical models of waiting in line provide a rich array of decision contexts that utilize non-linear functions to calculate critical performance measures. Queues is the British word for waiting lines, and they are an ever present element of all societies. The chapter's first context explores how to reduce the average waiting time for people standing in line to buy tickets to a show. The second example assesses the impact of merging small rural post offices. The final example evaluates two alternative layouts of airport security screening stations. The nonlinear functions presented in this chapter include higher order polynomials as well complex exponential functions.

Applied Mathematics Practices for the 21st Century AMP21

The Common Core State Standards Initiative is the result of an effort at the state level coordinated by the National Governors Association Center for Best Practices (NGA Center) and the Council of Chief State School Officers (CCSSO). The most provocative aspect of the document is the Standards for Mathematical Practice. Six of the eight standards for Mathematical Practice require that students:

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Look for and make use of structure.

Making sense of problems and solving them is the essential characteristic of this textbook . Mathematical modeling is typically the vehicle by which OR problems are conceptualized and explored. Moreover, OR practitioners use technological tools to obtain solutions to problems and perform sensitivity analyses. Finally, sensitivity analysis, the quintessential form of explaining and interpreting results, is a clear demonstration of understanding both the nature of the problem *and* its solution.

A New Way to Learn Mathematics

We created AMP21 to develop mathematical curriculum that directly address the above standards. On the pages that follow, you will be introduced to a wide range of real world problems that operations researchers solve every day. While real world problems can be difficult to solve due to their enormous size, in many cases, their solution depends on mathematics that you have already learned in your previous high school mathematics courses. So how, you may wonder, will this course be different?

This new course in the mathematics of operations research for high school students is different because it is applications-based and problem-driven. This means that the applications of the mathematics will be upfront, not at the end of the chapter and that the key ideas will be developed within the sorts of problems that gave them birth. The applications you will see focus around making decisions in business, government, or your own personal life. In addition, mathematics is not a spectator sport. Therefore, in the text, many questions are asked but not answered. Instead, you will provide the answers. Studying mathematics in this way may require you to develop a new mindset about what mathematics is, how it can be used, and the best way to learn it.

What is Operations Research and Business Analytics?

Operations research (OR) is a scientific way to analyze problems, make decisions, and improve processes. OR professionals try to provide a sound basis for decision-making. These decisions may focus on day-to-day operations that arise in a manufacturing plant. Or, they may involve long-range issues such as designing new environmental regulations or establishing minimum prison sentence guidelines.

Operations researchers attempt to understand the structure of complex situations. They develop mathematical and computer models of a system of people, machines, and procedures. If you have ever played the game Sim City, you have manipulated a computer model. Operations researchers often use numerical, algebraic, and statistical techniques to model the decision context. Then they manipulate their models to study the behavior of the system. They use this understanding to predict how the system will behave under different rules and policies to improve system performance.

Unlike most disciplines, we can point to specific events that mark the birth of operations research. OR was born in the years just prior to World War II. The British anticipated an air war with Germany. In 1937, they began to test radar. By 1938, they were studying how to use the information radar provided to direct the operations of their fighter planes.

Until this time, the word experiment usually meant a scientist carrying out a controlled experiment in a laboratory. In contrast, this radar-fighter plane project used a multidisciplinary team of scientists. They studied actual operating conditions in the field instead of in the laboratory. They then designed experiments in the field of operations, and the new term "operations research" was born. Their goal was to understand the operation of the complete system of equipment, people, and environmental conditions (e.g. weather, nighttime). Then they tried to improve the total system's performance. Their work was an important factor in winning the air war in Battle of Britain. OR eventually spread to all of the military services. Several of the leaders of this effort eventually won Nobel Prizes in their original fields of study.

All branches of the US Armed Forces during WWII formed similar groups of interdisciplinary scientists. These groups worked to protect naval convoys, search for enemy convoys, enhance anti-submarine warfare and improve the effectiveness of bombers. To do so, they collected data by directly observing operations. Then they built a mathematical model of the system. Next, they used the model to recommend improvements. Finally, they obtained feedback on the impact of the changes. Today, every branch of the military has its own operations research group. These OR groups include both military and civilian personnel. They play a key role in long-term strategy and weapons development. They also direct the operation of actions such as Operation Desert Storm. In addition, the National Security Agency has its own Center for Operations Research.

In the 1950s, national professional organizations were formed. These organizations published research journals and universities added OR departments. All of this raised operations research to the level of a profession. The leading professional organization is INFORMS (Institute for Operations Research and the Management Sciences). There are also operations research societies all across the globe. With regard to formal education, operations research became and remains one of the core competencies of the field of Industrial Engineering (IE). At the graduate level there is significant overlap between IE and OR. In business schools, OR generally falls within the domain of operations management or management science. Most mathematics departments also offer introductory OR courses at the junior or senior undergraduate level.

The use of OR expanded beyond the military to include other government organizations and private companies. The petroleum and chemical industries were early users of OR. They improved the performance of plants, developed natural resources and planned strategy. In the 1990s, OR models were critical enablers for multinational companies to become integrated global planners of facility operations and resource management.

In the last decade the field of operations research has gained added traction under the buzzword analytics. Numerous organizations have created analytics groups. A simple internet search yields: Google Analytics, Twitter Analytics, Pinterest Analytics, IBM Analytics, etc. Each spring INFORMS offers a Business Analytics Conference as well as special analytics conferences in areas such as healthcare.

Today, operations research and business analytics play important roles in industry and government as in:

- Airline, hospitality and entertainment industry – scheduling planes cruise ships, managing the capital investment, pricing tickets, taking reservations
- Pharmaceutical industry – managing research and development and designing sales territories;
- Delivery services – planning routes and developing pricing strategies
- Financial services – credit scoring, marketing, and internal operations
- Internet and marketing – managing the traffic to websites around the globe, tracking customers to target marketing programs
- Healthcare – hospital and healthcare clinic management, control of epidemics, effectiveness of procedures and causes of
- Commodities and lumber industry – managing mining, growing forests and cutting timber
- Local government – deploying emergency services
- Policy studies and regulation – environmental pollution, air traffic safety, AIDS, and criminal justice policy.

CHAPTER 1

Make Hard Decisions—Multi-Criteria Decision Making (MCDM)

Section 1.0: Introduction to Making Hard Decisions

We all face decisions in our jobs, in our communities, and in our personal lives. For example,

- Where should a new airport, manufacturing plant, power plant, or health care clinic be located?
- Which college should I attend, or which job should I accept?
- Which car, house, computer, stereo, or health insurance plan should I buy?
- Which supplier or building contractor should I hire?

Decisions such as these involve comparing alternatives that have strengths or weaknesses with regard to multiple objectives of interest to the decision. For example, your criteria in buying health insurance might be to minimize cost *and* maximize protection. Sometimes these multiple criteria get in each other's way.
Multi-criteria decision making (MCDM) is used when one needs to make a hard decision with many criteria. In this chapter, you will see one form of multi-criteria decision making. The method introduced in this chapter is a structured methodology designed to handle the tradeoffs among multiple criteria.

A Little History

One of the first applications of this method of MCDM involved the study of possible locations for a new airport in Mexico City in the early 1970s. The criteria considered included cost, capacity, access time to the airport, safety, social disruption, and noise pollution.

The problems in this chapter use the steps of multi-criteria decision making to make hard decisions. MCDM is a systematic approach to quantify an individual's preferences. Measures of interest are rescaled to numerical values on a 0–1 scale, with 0 representing the worst value of the measure and 1 representing the best. This allows the direct comparison of many diverse measures. In other words, with the right tool, it really is possible to compare apples to oranges! The result of this process is an evaluation of the alternatives in a rank order that reflects the decision makers' preferences.

For example, individuals, college sports teams, Master's degree programs, or even hospitals can be ranked in terms of their performance on many diverse measures. Another example is the Bowl Championship Series (BCS) in college football that attempts to identify the two best college football teams in the United States to play in a national championship bowl game. This process has reduced, but not eliminated, the annual end-of-year arguments as to which college should be crowned national champion.

Section 1.1: Choosing a Wireless Plan

Choosing a wireless plan is an important decision for many people. In fact, most teenagers own smart phones. When choosing a wireless plan, there are many factors to consider.

Q1. What factors would you consider if you were choosing a wireless plan?

In this chapter, you will develop a process for making important decisions, such as choosing a wireless plan, with many competing features. Before doing so, you will complete an opening activity.

1.1.1 Opening Activity

In this activity, you will make a decision about what wireless plan you would choose if you were considering a new plan. To do so, complete the following steps:

1. Make a list of possible wireless plans that you would consider using.
2. Collect data on each of these plans that you would find useful in making a decision.
3. Choose one of the plans based on your data.
4. Explain why you chose this plan over the others.

Q2. What possible issues do you foresee with using these steps to choose a wireless plan?

In the following sections, the steps of the MCDM process will be explained in the context of a high school student and her friend helping her parents to choose a wireless plan. Isabelle Nueva needs to help her mother and father decide on the best wireless plan to buy for their family. She and her friend, Angelo Franco, will use the MCDM process they learned in their math class to help her parents make this decision. Follow along with Isabelle as she and Angelo use the MCDM process to make this decision.

1.1.2 Identify Criteria and Measures

The first thing they do is identify the **criteria** of a wireless plan that were important to Isabelle's family. From discussions she had with her mother and father, Isabelle knew that the criteria that were important to them were cost, contract features, and phone service.

Q3. If you were choosing a wireless plan, what criteria would be important to you?

Isabelle and Angelo know that they need to find at least one way to **measure** each of the criteria. They decide to measure the Cost criterion using Monthly Charge, Monthly Access Fee, and Overage Fee. They decide to measure the Contract Features criterion using Number of GB of Data per Month, Rollover Data, and Contract Length. The measure of Phone Service is defined as Quality of Service. Each criterion and its measures are provided in Table 1.1.1.

Criteria	Measures
Cost	Monthly Charge
	Access Fee per Line
	Overage Fee ($/GB)
Contract Features	Data Plan
	Rollover Data
	Contract Length
Phone Service	Quality of Service

Table 1.1.1: Criteria and measures for choosing a wireless plan

Q4. How would you measure each of your criteria?

The value of three measures—the Monthly Charge, Overage Fee, and Access Fee per Line—could be any numerical amount within a reasonable range. These are examples of **continuous measures**. That is, these measures can take on any numerical value within a range.

Isabelle and Angelo decide that the data they collected for the other three measures can be grouped into a finite number of categories. All of the plans they looked at before focusing on just three plans, had values that were multiples of 2.5 GB. They ranged from a low of 7.5 GB to a high of 15 GB. Thus, they decided to treat this as a **categorical measure** with only four possible values for Data Plan: 7.5, 10, 12.5, and 15 GB.

To obtain data on Quality of Service they decide to use ratings from a consumer magazine. The magazine considered dropped or disconnected calls, static and interference, and voice distortion to rate the quality of service. Isabelle and Angelo decide to only consider plans the magazine rated “Good”, “Very Good”, or “Excellent”. Therefore, this measure has three categories.

Another categorical measure is Contract Length—the shortest time a customer must remain with a particular plan to avoid paying a fee to cancel the service. Isabelle’s parents were concerned about being locked into a plan for a long period of time. The plans under consideration have only three different Contract Lengths (0, 1 year, and 2 years). All plans they investigate seem to use one of these. Thus, the Contract Length measure has three possible values. The categorical measures and their possible values are provided in Table 1.1.2.

Q5. Of the measures you listed in Q4, which are continuous and which should be treated as categorical?

Q6. Create a table similar to Table 1.1.2 for your categorical measures identified in the previous question. In order to do this, you will need to research possible wireless plans. What sort of research would you need to do?

Categorical Measure	Categorical Values (from best to worst)
Data Plan	15 GB 12.5 GB 10 GB 7.5 GB
Rollover Data	Yes No
Quality of Service	Excellent Very Good Good
Contract Length	0 1 year 2 years

Table 1.1.2: Categorical variables with categories and numeric values

1.1.3 Collect Data

Isabelle's parents were considering three wireless plans: Trot, UST&T, and Horizon. Isabelle and Angelo collected the data they need to help her parents make their decision. The first data they collected were the basic monthly fees that appear in Table 1.1.3.

Angelo and Isabelle discussed the impact of the monthly access fee on the family's cost. The Nueva family planned to initially sign up for four lines, one each for the parents and their two older teenagers. Angelo suggested that instead of two measures, these data should be combined into one measure, Total Monthly Charge. This is calculated by multiplying the per line fee by the number of lines and adding it to the base monthly fee. With this calculation, the monthly fee would be $180 for Trot. The monthly fee for UST&T would be $250. Lastly, Horizon would cost $220 per month. However, Isabelle raised the possibility that her youngest brother who is in middle school might be given a fifth line. However, after some thought, they both agreed the cost of a fifth line should not be included in the decision analysis for now.

	Plan		
	Trot	**UST&T**	**Horizon**
Base Monthly Charge ($)	100	130	20
Monthly Access Fee ($/line)	20	30	50
Total Monthly Charge for Four Lines ($)	180	250	220

Table 1.1.3: Wireless plans monthly cost

The other data they collected about the various plans are included in Table 1.1.4 alongside the monthly cost of four lines.

Plan	Trot	UST&T	Horizon
Total Monthly Charge ($)	180	250	220
Overage Fee ($/GB)	50	15	10
Contract Length	2 years	1 years	None
Data Plan (GB/month)	10	15	7.5
Rollover Data	No	Yes	No
Quality of Service	Excellent	Very Good	Good

Table 1.1.4: Isabelle and Angelo's wireless plan data

Q7. Create a table similar to Table 1.1.4 for your wireless plan data.

1.1.4 Find the Range of Each Measure

Next, Isabelle and Angelo specify a range for each measure. They first specify the range for the two continuous measures (Total Monthly Charge and Overage Fee per GB). For each of these measures, they decide to use the range of the actual data they collected. That is, for Total Monthly Charge, the range was $180 to $250. The range Overage Fee was $10 to $50. For each of the categorical measures, Isabelle and Angelo simply list the two extreme values for each category. The **scale ranges** for each of Isabelle and Angelo's measures are given in Table 1.1.5.

Measure	Scale range
Total Monthly Charge	$180 to $250
Overage Fee per GB	$10 to $50
Contract Length	0 to 2 years
Data Plan	7.5 GB to 15 GB
Rollover Data	Yes or No
Quality of Service	Good to Excellent

Table 1.1.5: Ranges of each measure

Q8. Specify the ranges for each of your measures, and create a table similar to Table 1.1.5.

1.1.5 Rescale Data on All Continuous Measures to a Common Unit

It would be difficult to compare the three plans using these raw data. For example, how would one compare a $10 difference in the monthly service charge to a one-year difference in minimum contract length? In order to avoid such problems, operations researchers rescale the raw data of each measure to common unit values between zero and one. This creates a **common unit** that varies from zero to one for each measure. Zero always represents the worst value and one the best value for each measure.

For both of the continuous measures, Isabelle and Angelo use a **proportional scale** to assign a score to intermediate values. For example, the range for the Total Monthly Charge measure is $180 to $250. The smallest possible value here is the best option. Since the value one represents the best option, $180 is converted to a common unit value of one. Similarly, the largest possible value of the monthly service charge is the worst option. Thus, $250 converted to zero. That is,

$\$180 \rightarrow 1$
$\$250 \rightarrow 0$

Next, Isabelle and Angelo convert the price of Horizon's plan to a common unit value. They must decide what $220 should be converted to when it is compared to the best and worst values for Total Monthly Charge. The graph in Figure 1.1.1 illustrates this.

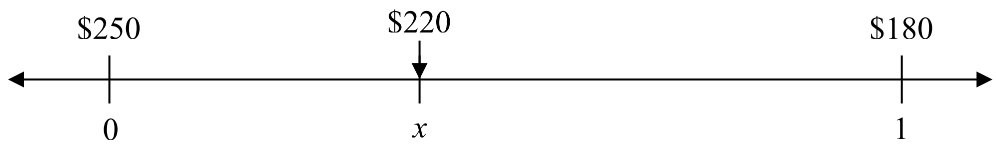

Figure 1.1.1: Determining the common unit values for the Total Monthly Charge measure

Q9. What do you think $220 should be converted to?

Q10. Is $220 closer to the best or the worst option?

Q11. How far is $220 from the best option? How far from the worst?

Isabelle and Angelo solve a proportion to arrive at the common unit value for the Total Monthly Charge of $220. To find the common unit value for $220 using proportions, Isabelle and Angelo write two equivalent fractions of the form $\frac{\text{part}}{\text{whole}}$. Figure 1.1.2 illustrates this.

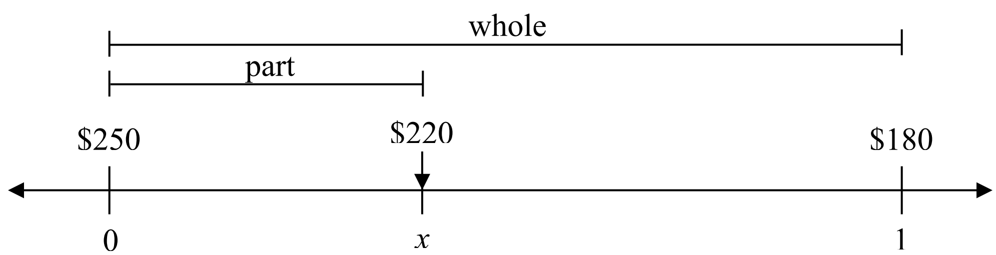

Figure 1.1.2: Determining the proportion to find the common unit values

In the first fraction, the "part" refers to the distance between $250 and $220, and the "whole" refers to the distance between $250 and $180. In the second fraction, the "part" refers to the distance between 0 and x, and the "whole" refers to the distance between 0 and 1. As can be seen in Figure 1.1.2, these two fractions are equivalent.

Isabelle and Angelo solve for the unknown in the equivalent fractions, using absolute value to find the distance between two values.

$$\frac{|220-250|}{|180-250|} = \frac{|x-0|}{|1-0|}$$

$$\frac{30}{70} = \frac{x}{1}$$

$$0.42 = x$$

Therefore, the raw value \$220 is converted to the common unit value 0.42.

Notice, each time these equivalent fractions are developed, the fraction on the right will always be:

$$\frac{|x-0|}{|1-0|}=\frac{x}{1}=x$$

Therefore, there is no need to write the entire fraction. Simply x can be used instead.

Q12. What other ways could Isabelle and Angelo use to calculate the common unit value for \$220?

Q13. Find the common unit values for the Overage Fee per GB measure.

1.1.6 Rescale Each Categorical Measure to a Common Unit

For the four categorical measures, Isabelle and Angelo assign a common unit value of zero to the worst option and one to the best option. For the Rollover Data measure, the only possible values are "yes" and "no". Yes was assigned a one, because is preferable; and no was assigned zero, because it is the worse value. When there was something between the best and worst values, Isabelle and Angelo discussed what to assign the intermediate values. With regard to Quality of Service, they simply assigned the one intermediate value, "very good," a score of 0.5. They used analogous reasoning for the two intermediate values of the Data Plan measure. They assigned common units proportionately: 10 GB was 0.33 and 12.5 was 0.67. However, they knew that Isabelle's parents really liked the idea of not being tied into a contract. A one-year contract was not much better than a two-year contract. They therefore assigned just 0.25 to a one-year contract. These conversions are summarized in Table 1.1.6.

Categorical Measure	Categorical Values	Common Units
Contract Length	0	1
	1 year	0.25
	2 years	0
Data Plan	15	1
	12.5	0.67
	10	0.33
	7.5	0
Rollover Data	Yes	1
	No	0
Quality of Service	Excellent	1
	Very Good	0.5
	Good	0

Table 1.1.6: Common unit values for the categorical measures

Isabelle and Angelo use the relationships developed above to convert the data for each plan into values between zero and one. The results of this conversion are presented in Table 1.1.7.

Plan	Trot	UST&T	Horizon
Total Monthly Charge ($)	1	0	0.43
Overage Fee ($/GB)	0	0.88	1
Contract Length	0	0.25	1
Data Plan (GB/month)	0.33	1	0
Rollover Data	0	1	0
Quality of Service	1	0.5	0
Total Points	**2.33**	**3.63**	**2.43**
Average Points	**0.39**	**0.61**	**0.41**

Table 1.1.7: Wireless plan data converted to a common unit

When Isabelle and Angelo looked at these results, they noticed that each plan received the top common unit value of one on two of the measures. They also noticed that each plan received at least one common unit value of zero. Therefore, it is not obvious to them which plan they should choose.

Q14. Based on the common unit values, which plan do you think Isabelle should recommend to her parents?

Angelo thinks they should use the total of all of the common units to get a total score for each plan. The totals are also listed in Table 1.1.7. Isabelle thinks it will be more meaningful to compute the average common unit scores for each plan. To do so, she divided the total score for each plan by six (the total number of measures and therefore the highest possible score). The averages she obtained are given in the bottom row of Table 1.1.7.

Q15. Do you think it makes more sense to use the sum or the average to make a decision?

Q16. Based on the total and average scores, which plan do you think Isabelle should recommend to her parents? Why?

Q17. What are some reasons why Isabelle may not recommend Horizon to her parents?

Q18. What are some reasons why Isabelle may think Trot would be a better choice for her parents?

Q19. What are some reasons why Isabelle may think UST&T would be a better choice for her parents?

Q20. Calculate the total scores and the average scores for each of your wireless plans.
   a. Based on these values, which plan would you choose?
   b. What are some reasons why these plans may not be the best choice for you?
   c. Was this plan what you expected to choose based on the opening activity? Why or why not?

Whether they use the sum or the average, Isabelle and Angelo realize that each plan has something in its favor. They wonder how to reach a decision. Then Isabelle remembers that her parents were really worried about the Total Monthly Charge, and not as worried about Contract Length. They decide that they need a system that does not treat all of the measures as equally important, as the sum and average do. They need a system that weights each measure according to how important it is to Isabelle's parents.

1.1.6 Conduct an Interview to Calculate Weights

In order to learn how important each measure is to her parents, Isabelle and Angelo decide to interview them. They want to learn which measure Isabelle's parents believe is most important to them. To do so, the parents will need to look closely at the most preferred value and least preferred value for each measure. Angelo and Isabelle explore with her parents how Isabelle's parents would rank order the six measure ranges. Mr. and Mrs. Nueva decide that the difference between the highest and lowest monthly payments was most important to them. The difference between lowest and highest is $70 per month; this is substantial. Therefore, they rank the Total Monthly Charge measure number one.

They knew their teenagers wanted to use their smart phones to download large files. They, therefore, rank Data Plan as the second most important measure. The Nueva's rated the Quality of Service as the third most important measure. They might have ranked it higher if the scale included poor service. However, since the minimum was "good," they were comfortable ranking it third most important. They really liked that Horizon offered a plan with no contract and therefore listed Contract Length as fourth. They were confident their children would strive to live within the monthly GB of data budget. However, they feared every once and a while they would lose track. In that case they could be shocked with a huge overage fee; they ranked Overage Fee fifth. They assumed their children would rarely have GBs of data to rollover into the next month. This measure was ranked last.

Table 1.1.8 shows their rank-ordering of the measures. For example, Total Monthly Charge is the most important measure to Isabelle's parents and Rollover Data is the least important. This table also includes the least and the most preferred values for each measure.

Measure	Least Preferred Value	Most Preferred Value	Rank
Total Monthly Charge ($)	250	180	1
Overage Fee ($/GB)	50	10	5
Contract Length	2 years	0	4
Data Plan (GB/month)	7.5	15	2
Rollover Data	No	Yes	6
Quality of Service	Good	Excellent	3

Table 1.1.8: Rank-order of the measures according to Isabelle's parents

Q21. Rank-order each of your measures.

Next, Isabelle and Angelo ask her parents to assign points to each measure to better capture the magnitude of the differences between two rankings. To make their decision-making model even more useful, they want a sense of how much more important one measure is than another. For example, if one measure is twice as important as another, then the assigned points should be twice as much for the higher ranked measure.

Isabelle and Angelo ask Mr. and Mrs. Nueva to assign 100 points to Total Monthly Charge, the measure they ranked number one. Then, they ask them to assign a number of points less than 100 to the second-ranked measure, Data Plan. In doing so, they ask Isabelle's parents to pick a number that reflects how important Data Plan is compared to the Total Monthly Charge.

Mr. and Mrs. Nueva decide to assign 90 points to Family Data, because they know their children like to download large files. It is almost as important as the Total Monthly Charge. Quality of service was also important to them and only slightly less important than Data Plan. This was given 80 points. Although they liked not having a contract, it really was far less important than the first three measures. They assigned it 40 points, or half the weight of Quality of Service. The high overage fee was a risk they thought they could manage and gave it only 20 points. They did not think there was much value to their family of Rollover Data. They assigned it only 10 points. The Nueva's preferences are summarized in Table 1.1.9.

Measure	Least Preferred Value	Most Preferred Value	Rank	Points
Total Monthly Charge ($)	250	180	1	100
Overage Fee ($/GB)	50	10	5	20
Contract Length	2 years	0	4	40
Data Plan (GB/month)	7.5	15	2	90
Rollover Data	No	Yes	6	10
Quality of Service	Good	Excellent	3	80

Table 1.1.9: Points assigned to each of the measures

Q22. Assign points to each of your measures, and create a table similar to Table 1.1.9.

Now, Isabelle and Angelo total all of the assigned points and obtain 340. Then, they divide the point assignment for each measure by that total. This number is the **weight** of that measure. For example, monthly charge was assigned 100 points. Thus, the weight of this measure is:

$$\frac{100}{340} = 0.29$$

One way of interpreting the weight of 0.29 for Total Monthly Charge is that 29% of the final decision will be based on this measure. The results of Isabelle and Angelo's interview of her parents are summarized in Table 1.1.10.

Measure	Least Preferred Value	Most Preferred Value	Rank	Points	Weight
Total Monthly Charge ($)	250	180	1	100	0.29
Overage Fee ($/GB)	50	10	5	20	0.06
Contract Length	2 years	0	4	40	0.12
Family Data (GB/month)	7.5	15	2	90	0.26
Rollover Data	No	Yes	6	10	0.03
Quality of Service	Good	Excellent	3	80	0.24

Table 1.1.10: Calculated weight for each measure

Q23. What measure has the largest weight? Which has the smallest?

Q24. What is the ratio of the largest weight to the smallest weight?

Q25. What should this ratio mean in the context of the decision?

Q26. Assign points to each of your measures, and create a table similar to Table 1.1.10.

1.1.7 Calculate Total Scores

Now, Isabelle and Angelo calculate a **total score** for each plan. The total score is an example of a **weighted average**. They multiply each common unit value from Table 1.1.7 by the corresponding weight from Table 1.1.10. Then for each plan, they sum those six products together to get the total score. The data from these two tables are placed side-by-side in Table 1.1.11. The results of these computations are given in Table 1.1.12. Notice that this weighted average captures how important the various measures are to Isabelle's parents.

Measure	**Weight**	Trot	UST&T	Horizon
Total Monthly Charge ($)	0.29	1	0	0.43
Overage Fee ($/GB)	0.06	0	0.88	1
Contract Length	0.12	0	0.25	1
Family Data (GB/month)	0.26	0.33	1	0
Rollover Data	0.03	0	1	0
Quality of Service	0.24	1	0.5	0

Table 1.1.11: Measure weights and wireless plan scores

Measure	Weight	Trot	UST&T	Horizon
Total Monthly Charge ($)	0.29	1 × 0.29 = 0.29	0 × 0.29 = 0	0.43 × 0.29 = 0.13
Overage Fee ($/GB)	0.06	0 × 0.06 = 0	0.88 × 0.06 = 0.05	1 × 0.06 = 0.06
Contract Length	0.12	0 × 0.12 = 0	0.25 × 0.12 = 0.03	1 × 0.12 = 0.12
Family Data (GB/month)	0.26	0.33 × 0.26 = 0.09	1 × 0.26 = 0.26	0 × 0.26 = 0
Rollover Data	0.03	0 × 0.03 = 0	1 × 0.03 = 0.03	0 × 0.03 = 0
Quality of Service	0.24	1 × 0.24 = 0.24	0.5 × 0.26 = 0.13	0 × 0.26 = 0
Wireless Plan's Total Score		**0.62**	**0.50**	**0.31**

Table 1.1.12: A weighted total score is computed for each plan.

Q27. Multiply the common unit values by the corresponding weights for each of your plans, and create a table similar to Table 1.1.12.

Q28. Would everyone's score results lead to the same preferred choice? Explain.

1.1.8 Determine Strengths/Weaknesses and Make Final Decision

Trot is clearly the preferred plan. UST&T is a distant second. Isabelle and Angelo decide to closely examine the results. They clearly do not produce the same results as the sum or average methods.

Q29. For which measures does Trot have a higher weighted score than UST&T? For which does UST&T outscore Trot?

When Isabelle and Angelo compare Trot with UST&T, they see that Trot had higher weighted scores for the first- and third-ranked measures, Total Monthly Cost and Quality of Service. UST&T scored higher on the other four measures. However, the magnitude of the difference for measures ranked four, five, and six was always small. In each case the difference was only 0.03 or less. These could not overcome the advantage Trot had on Total Monthly Cost, the highest ranked measure. Their weighting system did what it was supposed to do; it took into account Mr. and Mrs. Nueva's preferences. They decide to recommend the Trot plan to Isabelle's parents.

1.1.9 Alternative Trot Plan

Trot recently announced an alternative that comes with a larger Data Plan. This plan comes with 12.5 GB of data per month. It also costs $15 a month more. The two plans are compared in Table 1.1.13.

Plan	Old Trot	New Trot
Total Monthly Charge ($)	180	195
Overage Fee ($/GB)	50	50
Contract Length	2 years	2 years
Data Plan (GB/month)	10	12.5
Rollover Data	No	No
Quality of Service	Excellent	Excellent

Table 1.1.13: Alternative Trot Plan

To compare the two plans, Isabelle and Angelo must first convert the new values to common units between zero and one. Then they will need to multiply the values by their corresponding weights.

Q30. What is the common unit value for the monthly charge of $195?

Q31. What is the common unit value for the Data Plan, 12.5 GB per month?

Q32. Should Isabelle and Angelo recommend to Mr. and Mrs. Nueva that they adopt the new Trot plan?

1.1.10 Summary

In this problem, Isabelle and Angelo wanted to help Isabelle's parents choose a wireless plan. They completed the following steps:

1. Identify criteria and measures
2. Collect data
3. Find the range of each measure
4. Rescale each measure to a common unit

After completing these steps, Isabelle and Angelo found the total score and the average score for each wireless plan. However, they noticed that these values treated all measures under consideration as being equally important. This was not a reasonable way to make a decision. They needed a way to weight some measures more than others, because Isabelle's parents were more concerned about the cost of the plan than anything else.

In order to take Mr. and Mrs. Nueva's preferences regarding a wireless plan into account, Isabelle and Angelo completed four additional steps:

5. Conduct an interview to rank order measures and assign points

6. Calculate the weight of each measure
7. Calculate a total score for each alternative
8. Interpret results

This eight-step process will be applied in the next two sections and in the homework problems to make slightly more complicated decisions. This process is also a life skill, because you may find it useful to help you make important decisions in your future.

Section 1.2: Enrique Ramirez Chooses a College

Enrique Ramirez has been accepted at four colleges: Canisius College in Buffalo, NY; Clark University in Worcester, MA; Drexel University in Philadelphia, PA; and Suffolk University in Boston, MA. Now he must decide which one to attend.

Enrique asks his friend Anna for help. Enrique and Anna realize that there are many different issues to consider when making this decision. They also realize that the issues of interest to Enrique and their relative importance are not the same as those for Anna.

To make this decision, Enrique, with the help of Anna, follows the steps of MCDM that were presented in the previous section. These steps are given in Table 1.2.1.

General Steps	Descriptions for this Particular Decision
1. Identify Criteria and Measures	First, generate a list containing general criteria that are important when choosing a college. These criteria will be broad in nature and will be based on objective and subjective goals. Next, specify at least one measure for each criterion.
2. Collect Data	For each college, collect the data for each measure.
3. Find the Range of Each Measure	Specify a reasonable scale for each measure.
4. Rescale Each Measure to a Common Unit	Rescale each measure to common units from 0 to 1, with 0 being the worst alternative and 1 being the best alternative.
5. Conduct an Interview to Calculate Weights	With the help of an interviewer, rank-order the measures, assign points from 0 to 100 to each measure, and calculate a proportional weight between 0 and 1 for each measure.
6. Calculate Total Scores	Calculate a total weighted score for each college. These weights will yield a ranking of the colleges, allowing you to identify the best option based on your preferences.
7. Interpret Results	Review the results to understand the strengths and weaknesses of your top alternatives before finalizing your decision.

Table 1.2.1: Steps of MCDM

1.2.1 Identify Criteria and Measures

With Anna's help, Enrique decides that academics, cost, location, and social life are the criteria most critical in his choice of a school. Next, Enrique and Anna take his list of four criteria and specify two or three measures for each criterion. His criteria and measures are given in Table 1.2.2.

Criteria	Measures
Academics	Average SAT Score (based on last year's freshman class)
	U.S. News & World Report Ranking
Cost	Room & Board (annual)
	Tuition (annual)
Location	Average Daily High Temperature
	Nearness to Home
Social Life	Athletics
	Reputation
	Size

Table 1.2.2: Enrique's criteria and measures

1.2.2 Collect Data

For each measure, Enrique and Anna collect data, which is listed in Table 1.2.3. Some of the measures are naturally categorical. For example, *U.S. News & World Report* ranks schools into four categories:

1. Nationally ranked
2. Regionally ranked
3. Regionally tier 3
4. Regionally tier 4

Enrique and Anna divide Athletics into three categories:

1. Division 1
2. Division 2
3. Division 3

Similarly, they divide Reputation into three categories:

1. Seriously academic
2. Balanced academics and social life
3. Party school

The data for the remaining measures are numerical values. Enrique and Anna are able to find the average values for SAT score and daily high temperature and the exact values for room and board cost, tuition cost, nearness to home, and size.

Measure	Canisius	Clark	Drexel	Suffolk
Average SAT Score	1590	1750	1700	1480
U.S. News & World Report Ranking	22nd (regional)	91st (national)	109th (national)	Tier 3 (regional)
Room & Board	$10,150	$8,850	$12,135	$11,960
Tuition	$28,157	$33,900	$30,470	$25,850
Average Daily High Temperature	56°	56°	64°	59°
Nearness to Home	297 mi	157 mi	81 mi	191 mi
Athletics	Division 1	Division 3	Division 1	Division 3
Reputation	Balanced	Seriously academic	Seriously academic	Balanced
Size	3,300 students	2,175 students	12,348 students	4,985 students

Table 1.2.3: Raw data for Enrique's four schools

1.2.3 Find the Range of Each Measure

Next, Enrique and Anna choose an appropriate scale for each of the nine measures. Some of the measures are continuous (e.g., SAT score), while others are categorical (e.g., athletics).

For the Nearness to Home measure, Enrique believes that exact mileage is not important, but rather broad ranges of mileage better represent his concerns. Therefore, Enrique and Anna convert this measure from continuous to categorical.

Q1. Looking at Table 1.2.4, what other measure was converted from continuous to categorical?

Enrique and Anna also realize that the range of each scale is important. For example, the theoretical range of the average combined SAT score is 600–2400, but in actuality, the range of the average combined SAT score at the colleges Enrique is considering is 1480–1750, which is a much narrower range. Enrique and Anna decide that it is much more realistic to use a range that is close to the actual range.

Q2. In the previous section, the ranges for the continuous measures were simply the ranges of the data collected. In this section, the ranges are expanded slightly. For example, instead of the SAT range staying as 1480-1750, Enrique and Anna choose the range 1400-1800. Why might one prefer to use the ranges of the data collected? Why might one prefer to round the ranges?

Q3. Looking at Table 1.2.4, for what other measures do Enrique and Anna create realistic ranges? Do you agree with their ranges? Why or why not?

The type and scale range of each measure are given in Table 1.2.4.

Measure	Type	Scale range
Average SAT Score	Continuous	1400–1800
U.S. News & World Report Ranking	Categorical	Nationally Ranked
		Regionally Ranked
		Regionally Tier 3
		Regionally Tier 4
Room & Board	Continuous	\$8,000–\$14,000
Tuition	Continuous	\$25,000–\$35,000
Average Daily High Temperature	Continuous	50˚–70˚F
Nearness to Home	Categorical	Within 1 hr. Drive (50-100 mi)
		Within 4 hr. Drive (101–200 mi)
		Within a Day's Drive (201–300 mi)
Athletics	Categorical	Division 1
		Division 2
		Division 3
Reputation	Categorical	Seriously Academic
		Balanced Academics and Social Life
		Party School
Size	Categorical	Under 3,000 students
		3,001–6,000 students
		6,001–12,000 students
		Over 12,000 students

Table 1.2.4: Types and ranges of measures

Before continuing, Enrique and Anna convert the values of the categorical measures into the numerical values based on the ranges of each measure. The converted data for the categorical measures are given in Table 1.2.5.

Measure	Canisius	Clark	Drexel	Suffolk
Average SAT Score	1590	1750	1700	1480
U.S. News & World Report Ranking	2	1	1	3
Room & Board	\$10,150	\$8,850	\$12,135	\$11,960
Tuition	\$28,157	\$33,900	\$30,470	\$25,850
Average Daily High Temperature	56˚	56˚	64˚	59˚
Nearness to Home	3	2	1	2
Athletics	1	3	1	3
Reputation	2	1	1	2
Size	2	1	4	2

Table 1.2.5: Converted categorical data for Enrique's four schools

1.2.4 Rescale Each Measure to a Common Unit

Once Enrique and Anna choose appropriate scales for each of the measures, Anna reminds Enrique that if they compared the data in its current form, it would be like comparing apples to oranges. They decide to convert the data to common units. To do so, they assign 1 to the best value and 0 to the worst value in the range of each measure. Recall from the previous section, the method for determining intermediate values differs for continuous and categorical measures.

Converting Continuous Measures
For the continuous measures, Enrique and Anna use a proportional scale. For example, the Average SAT Score at Canisius is 1590. The range for this measure is 1400–1800, so 1400 (the least desirable score) should be converted to 0 and 1800 (the most desirable score) to 1. But what should the proportional value for Canisius be?

$$1800 \to 1$$
$$1590 \to x$$
$$1400 \to 0$$

Figure 1.2.1 illustrates this example.

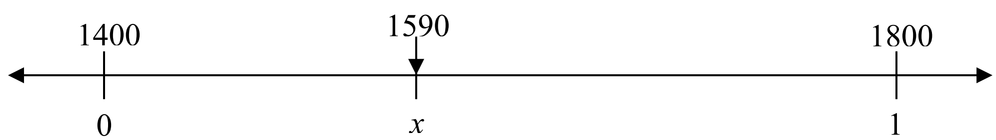

Figure 1.2.1: Determining the common unit values for the SAT scores measure

Using the same method as in the previous section, Enrique and Anna solve a proportion to find x.

$$\frac{|1590-1400|}{|1800-1400|}=\frac{|x-0|}{|1-0|}$$
$$\frac{190}{400}=\frac{x}{1}$$
$$x = 0.475$$

Therefore, an average SAT score of 1590 is converted to a common unit value of 0.475. Enrique and Anna decide to use proportional common units for each of the measures that have a continuous scale.

Converting Categorical Measures: Proportional or Non-proportional Scale
For the categorical measures, Enrique and Anna begin by assigning the best value a 1 and the worst value a 0. Then, Enrique and Anna decide how to apportion the common units. In some cases, apportionment is proportional, while in other cases it is not. They decide to use proportional common units for Nearness to Home, Athletics, and Reputation.

On the other hand, Enrique feels that some categorical measures should not be apportioned proportionately. For example, Enrique and Anna decided that there is a big difference between being ranked nationally and regionally on the *U.S. News & World Report* Ranking measure. Thus, they decide to have the following common units:

Nationally Ranked → 1
Regionally Ranked → 0.5
Tier 3 → 0.25
Tier 4 → 0

Enrique prefers a smaller school. Therefore, he assigns the following common units:
Under 3,000 students → 1
3,000-6,000 students → 0.75
6,001-12,000 students → 0.25
Over 12,000 students → 0

Table 1.2.6 contains the results of Enrique and Anna's rescaling of each measure to common units.

Measure	**Canisius**	**Clark**	**Drexel**	**Suffolk**
Average SAT Score	0.475	0.875	0.750	0.200
U.S. News & World Report Ranking	0.50	1	1	0.25
Room & Board	0.642	0.858	0.311	0.340
Tuition	0.684	0.110	0.453	0.915
Average Daily High Temperature	0.30	0.30	0.70	0.45
Nearness to Home	1	0.5	0	0.5
Athletics	1	0	1	0
Reputation	1	0.5	0.5	1
Size	0.75	1	0	0.75

Table 1.2.6: Each measure rescaled to common units

Q4. From Table 1.2.3, Clark University has the highest average combined SAT score and the highest tuition. Why does it make sense in Table 1.2.6 that Clark has the highest common unit value on one of those measures, but the lowest common unit value on the other?

Q5. Looking at the Nearness to Home measure in Table 1.2.6, what was the most desirable distance to Enrique? What was least desirable?

Q6. Looking at the Athletics measure in Table 1.2.6, what was the most desirable division to Enrique? What was least desirable?

Q7. Looking at the Reputation measure in Table 1.2.6, what was the most desirable reputation to Enrique? What was least desirable?

To review, there are essentially three steps to rescale data to common units.

Step 1: Assign 1 to the best value in the range.
Assign 0 to the worst value in the range.

Step 2: For continuous data, assign intermediate scores proportionally:

$$\text{Scaled Score} = \frac{|\text{score} - \text{least preferred score}|}{|\text{most preferred score} - \text{least preferred score}|}$$

Step 3: For categorical data, assign intermediate scores proportionally or based on your own opinions and values.

1.2.5 Conduct an Interview to Calculate Weights

Next, Enrique and Anna assign weights to each of the measures to reflect the relative importance Enrique attaches to each of them. They decide Anna will interview Enrique. She makes observations to ensure that Enrique understands the measures he chose and the effects of the weights he assigns to each of them. As a reference tool during the interview, they create Table 1.2.7.

Anna: We have some measures and their ranges for making a decision about your college preference. Focus first on the column of least preferred values. Which one of the measures would you most want to increase from the least preferred value to its most preferred value? For example, is it more important to you to move the SAT score from 1400 to 1800 or to reduce tuition from $35,000 to $25,000?

Enrique: Lower the tuition!

Anna: Are you sure that lowering the tuition to $25,000 is the most important improvement in the whole list?

Enrique: Yes, so I think we should rank tuition number one.

Anna: Enrique, what would be the next most important measure to move from least preferred to most preferred?

Enrique: *U.S. News & World Report* ranking is important, so let's rank that second, and SAT score third.

They continue like this until each measure has been ranked, as shown in Table 1.2.7.

The next task is to subjectively assign points from 0 to 100 for each measure based on the rank order. The points assigned reflected the relative importance Enrique places on each measure. They continue the interview to assign these points. During this interview process, Anna encourages Enrique to think about the relative importance of moving between the best and worst values of the two measures being considered. This is seen in the next part of the interview.

Criterion	Measure	Least preferred	Most preferred	Rank order	Points (0–100)
Academics	Average SAT Score	1400	1800	3	85
	U.S. News & World Report Ranking	Tier 4	Nat'l. Rank	2	90
Cost	Room & Board	$14,000	$8,000	4	80
	Tuition	$35,000	$25,000	1	100
Location	Average Daily High Temperature	50° F	70° F	9	20
	Nearness to Home	Within 1 hr.	Within 1 day	6	60
Social life	Athletics	Div. 3	Div. 1	8	30
	Reputation	Party	Balanced	7	50
	Size	> 12,000	< 3,000	5	70
				Sum:	585

Table 1.2.7: Enrique's ranking and point assignment

Anna: Let's start by assigning 100 points to the tuition range, which you've ranked first. Now, you've ranked *U. S. News & World Report* rating second. How important is this rating, from worst to best, compared to reducing the cost of tuition from $35,000 to $25,000? If it's close, you should use a number close to 100.

Enrique: I think it's about 90% as important, so let's use 90 points for that one, and SAT scores are almost as important, so we'll use 85 points for that range.

Table 1.2.7 contains the rest of the points Enrique assigns to each of his measures.

The interview continues:

Anna: Enrique, what did you get for the total number of points for all your measures? Once you have the point total, you'll need to divide the points for each measure by this total to get the weight.

Enrique: I got 585 total points. Now I can calculate the individual measure weights.

The weights Enrique calculates appear in Table 1.2.8. These were calculated by dividing the points for a particular measure by the total points. For example, Average SAT Score has a point value of 85. So, the weight for this measure is:

$$\frac{85}{585} = 0.145 .$$

Criterion	Measure	Least preferred	Most preferred	Rank order	Points (0–100)	Weight (Points/Sum)
Academics	Average SAT Score	1400	1800	3	85	0.145
	U.S. News & World Report Ranking	Tier 4	Nat'l. Rank	2	90	0.154
Cost	Room & Board	$14,000	$8,000	4	80	0.137
	Tuition	$35,000	$25,000	1	100	0.171
Location	Average Daily High Temperature	50° F	70° F	9	20	0.034
	Nearness to Home	Within 1 hr.	Within 1 day	6	60	0.103
Social life	Athletics	Div. 3	Div. 1	8	30	0.051
	Reputation	Party	Balanced	7	50	0.085
	Size	> 12,000	< 3,000	5	70	0.120
				Sum:	585	1.000

Table 1.2.8: Enrique's assignment of weights to each measure

Next, Anna wants to ensure that Enrique has assigned an appropriate weight to each *criterion*. The interview continues.

Anna: Enrique, what is the total weight for each criterion?

Enrique: I get a total of 0.299 for academics, 0.308 for cost, 0.137 for location, and 0.256 for social life.

Anna: Which criterion has the greatest weight assigned to it?

Enrique: It looks like cost, with 0.308.

Anna: Are there criteria with similar weights?

Enrique: It looks like academics and cost are almost the same.

Anna: Are these the criteria you feel are the most important criteria for choosing a college, and do you think they're about the same in importance?

Enrique: I didn't realize I placed so much importance on academics.

Anna: What did you expect to happen?
Enrique: I thought social life would be at the top of the list!

Anna: Well, you gave athletics only 30 points, reputation 50 points, and size 70 points. Do you want to change anything?
Enrique: No, I really think academics and cost are most important.

1.2.6 Calculate Total Scores

Finally, Enrique and Anna calculate a total score for each school. They use the data from Table 1.2.6, where common units were computed, and the weights calculated in the last column of Table 1.2.8 to calculate a score for each school on each measure. In Table 1.2.9 below, Enrique has calculated the product of the weight, W, and the corresponding common unit, CU:

Score = $W \cdot CU$

For example, the common unit score for the average SAT score at Canisius College is 0.475, and the weight Enrique has assigned to average SAT score is 0.145. Multiplying these two numbers yields 0.069. This value appears opposite SAT score and below Canisius in Table 1.2.9. It is 10% of the total score for Canisius. The rest of the values in Table 1.2.9 are computed in the same way. Then, totaling the scores for each measure for each college yields the total scores that appear in the last row in Table 1.2.9 Now Enrique can see which of his college choices best suits his preferences.

Measure	Weight	Canisius	Clark	Drexel	Suffolk
Average SAT Score	0.145	$0.145 \cdot 0.475 = 0.069$	0.127	0.109	0.029
U.S. News & World Report Ranking	0.154	0.077	0.154	0.154	0.038
Room & Board	0.137	0.088	0.117	0.043	0.046
Tuition	0.171	0.117	0.019	0.077	0.156
Average Daily High Temperature	0.034	0.010	0.010	0.024	0.015
Nearness to Home	0.103	0.103	0.051	0	0.051
Athletics	0.051	0.051	0	0.051	0
Reputation	0.085	0.085	0.043	0.043	0.085
Size	0.120	0.090	0.120	0	0.090
Total Score:	**1.000**	**0.690**	**0.641**	**0.501**	**0.512**

Table 1.2.9: Calculating the measure score and total scores of Enrique's schools

1.2.7 Interpreting the Results

Enrique reviews these results carefully. He notices that Drexel and Suffolk have scored much lower than his top-ranked choice, so he excludes them from further study. However, he decides to take a closer look at the relative strengths and weaknesses of Canisius, ranked first, and Clark, ranked second. There is only a 0.049 difference between the two, and he is not sure that it is enough evidence to make this critical life decision.

Q8. What are some reasons why Enrique may not choose Canisius, even though it was ranked first?

Q9. On many of the measures, Clark received better scores than Canisius. Why did Canisius end up having the higher total score?

Q10. Suppose Enrique was offered a scholarship at Clark for $5,000. How do you think this would affect Enrique's decision?

Section 1.3: Judy Purchases a Used Car

Judy is trying to decide which used car to purchase from among four possibilities: a 2006 Honda Civic Hybrid, a 2006 Toyota Prius, and a 2007 Nissan Versa that she has found at dealerships, as well as a 2005 Ford Focus that Judy's uncle Roger is trying to sell by himself. Judy asks her friend Dave to help her structure her thoughts in a consistent manner and to use the steps in the process of multi-criteria decision making (see Section 1.2 for a list of the steps).

With Dave's help, Judy decides that the criteria most important for her choice of a used car are minimizing total cost and maximizing condition, accessories, and aesthetics. They identify two measures for each criterion, as shown in Table 1.3.1.

Criterion	Measures
Total cost	Purchase price
	Miles per gallon, based on the EPA rating when new
Condition	Odometer reading
	Body condition
Accessories	Functional air conditioner and heater
	Sound system
Aesthetics	Color
	Body design

Table 1.3.1: Judy's list of criteria and measures

Judy and Dave collect data on each of the cars being considered. Their data appear in Table 1.3.2.

Measure	Honda Civic Hybrid	Toyota Prius	Ford Focus	Nissan Versa
Purchase Price	\$15,000	\$15,500	\$7,700	\$11,000
Miles per Gallon	43	46	25	33
Odometer Reading	85,000	80,000	95,000	65,000
Body Condition	Good	Good	Good	Excellent
Functional Air Conditioner and Heater	Both Work	Both Work	Both Work	Both Work
Sound System	Radio and CD Players	Radio and CD Players	Radio, CD, and MP3 Players	Radio and CD Players
Color	Red	Silver	Blue	White
Body Design	Sedan	Sedan	Wagon	Hatch-back

Table 1.3.2: Judy's data on four used cars

After collecting data and determining the scale range for each measure, Judy and Dave create Table 1.3.3. At this point, for each of the categorical measures, she assigned an integer value. For example, the five possible colors were given values from 1 for blue, the least preferred to 5 for the most preferred color. She created four categories for the sound system and numbered them from 1 to 4.

Measure	Scale range		Type
Purchase Price	$6,000–$16,000		Continuous
Miles per Gallon	20–50 mpg		Continuous
Odometer Reading	50,000–100,000 miles		Continuous
Body Condition	1	Fair	Categorical
	2	Good	
	3	Excellent	
Functional Air Conditioner and Heater	1	Neither works	Categorical
	2	Only one works	
	3	Both work	
Sound System	1	None	Categorical
	2	Radio only	
	3	Radio and CD player	
	4	Radio, CD, and MP3	
Color	1	Blue	Categorical
	2	Red	
	3	Silver	
	4	White	
	5	Black	
Body Design	1	Wagon	Categorical
	2	Hatchback	
	3	Sedan	

Table 1.3.3: The type and range for each of Judy's measures

Next, Judy and Dave convert the data for each car into the numerical values given in Table 1.3.4.

Measure	Honda Civic Hybrid	Toyota Prius	Ford Focus	Nissan Versa
Purchase Price	$15,000	$15,500	$7,700	$11,000
Miles per Gallon	43	46	25	33
Odometer Reading	85,000	80,000	95,000	65,000
Body Condition	2	2	2	3
Functional Air Conditioner and Heater	3	3	3	3
Sound System	3	3	4	3
Color	2	3	1	4
Body Design	3	3	1	2

Table 1.3.4: Converted categorical data for Judy's four cars

However, to determine a total score, each value must be rescaled to values between 0 and 1. Judy and Dave convert each continuous measure proportionally (on a scale from 0 to 1) and each categorical measure subjectively, based on Judy's preferences. Table 1.3.5 shows each common unit value. Use this table to answer the questions below.

Measure	Honda Civic Hybrid	Toyota Prius	Ford Focus	Nissan Versa
Purchase price	0.1	0.05	0.83	0.5
Miles per gallon	0.767	0.867	0.167	0.433
Odometer reading	0.3	0.4	0.1	0.7
Body condition	0.5	0.5	0.5	1
A/C and heater	1	1	1	1
Sound system	0.75	0.75	1	0.75
Color	0.25	.5	0	0.75
Body design	1	1	0	0.5

Table 1.3.5: Each measure rescaled to common units

Q1. Consider the continuous measures. Determine how Judy and Dave calculated the common unit values for:
  a. Purchase Price
  b. Miles per Gallon
  c. Odometer Reading

Q2. Now consider the categorical measures. Determine Judy's preferences for the following measures based on the information in Table 1.3.5:
  a. Body Condition
  b. Functional Air Conditioner and Heater
  c. Sound System
  d. Color
  e. Body Design

While the information given in Table 1.3.5 is informative, it does not take Judy's preferences into consideration because each measure is weighted equally.

Therefore, Dave interviews Judy to determine how they would weight each measure. The rank order, points, and weights that came out of this interview can be seen in Table 1.3.6. Use this table to answer the questions below.

Criteria	Measure	Least preferred	Most preferred	Rank order	Points (0–100)	Weight (Points/Sum)
Total cost	Purchase Price	$16,000	$6,000	1	100	0.2
	Miles per Gallon	20 mpg	50 mpg	2	95	0.19
Condition	Odometer Reading	100,000 mi	50,000 mi	5	60	0.12
	Body Condition	1 (fair)	3 (excellent)	3	75	0.15
Accessories	Functional Air Conditioner and Heater	1 (neither works)	3 (both work)	7	35	0.07
	Sound System	1 (none)	4 (radio, CD, MP3)	6	50	0.1
Aesthetics	Color	1 (blue)	5 (black)	8	10	0.02
	Body Design	1 (wagon)	3 (sedan)	3	75	0.15
				Sum =	500	1

Table 1.3.6: Judy's rank ordering, point assignment, and weight calculation for her measures

Q3. Which measure is most important to Judy? How do you know?

Q4. Which measure is least important to Judy? How do you know?

Q5. Which two measures have equal importance to Judy? How do you know?

Q6. How would you describe Judy's feelings towards Purchase Price versus her feelings towards Miles per Gallon?

Q7. How would you describe Judy's feelings towards Miles per Gallon versus her feelings towards Body Condition?

Q8. How were the weights calculated?

Q9. What criterion is most important to Judy? How do you know?

Finally, Judy and Dave calculate the total scores for each car, as shown in Tables 1.3.7 and 1.3.8. Use these tables to answer the questions below.

Measure	Weight	Honda Civic Hybrid	Toyota Prius	Ford Focus	Nissan Versa
Purchase price	0.2	0.1	0.05	0.83	0.5
Miles per gallon	0.19	0.767	0.867	0.167	0.433
Odometer reading	0.12	0.3	0.4	0.1	0.7
Body condition	0.15	0.5	0.5	0.5	1
A/C and heater	0.07	1	1	1	1
Sound system	0.1	0.75	0.75	1	0.75
Color	0.02	0.25	.5	0	0.75
Body design	0.15	1	1	0	0.5

Table 1.3.7: Judy's weights and common units for each measure

Measure	Honda Civic Hybrid	Toyota Prius	Ford Focus	Nissan Versa
Purchase price	0.02	0.01	0.166	0.1
Miles per gallon	0.1457	0.1647	0.0317	0.0823
Odometer reading	0.036	0.048	0.012	0.084
Body condition	0.075	0.075	0.075	0.15
A/C and heater	0.07	0.07	0.07	0.07
Sound system	0.075	0.075	0.1	0.075
Color	0.005	0.010	0.000	0.015
Body design	0.15	0.15	0	0.075
Total Score	**0.582**	**0.613**	**0.470**	**0.636**

Table 1.3.8: Judy's calculation of the measure subtotal score and total score for each used car

Q10. How were the scores for each measure calculated?

Q11. Looking at the total scores, which car would you recommend to Judy? Is this choice obvious?

Q12. What significant advantages does the Toyota Prius have over the Nissan Versa?

Q13. What significant advantages does the Nissan Versa have over the Toyota Prius?

When the numeric values are this close, the ultimate answer may be that the decision maker will be equally satisfied with either choice.

Q14. If you were choosing among these cars, which car would you choose? Was your choice impacted by the total scores calculated in this problem?

1.3.1 Use Excel to Calculate Scores

The spreadsheet format of Excel offers an ideal tool to calculate scores. The data in table 1.3.5 are the common unit scores for each car on each measure. These were input into EXCEL as seen in Figure 1.3.2. To determine the total score, for example, for the Honda, Judy will need to multiply the weights in column B by the common units in column C. Judy can carry out this computation in either of two ways. The simpler method involves using an EXCEL function named SUMPRODUCT. This command involves specifying the two sets of numbers that are to be multiplied and then summed. In this example we want to multiply the values in cells B3 through B10 by the corresponding values in Cells C3 to C10. In EXCEL you specify a range with a semicolon as in B3:B10 and as in C3:C10. The two ranges are separated by a comma in the command. The SUMPRODUCT command multiplies the value in B3 and by the value in C3, multiplies the value in B4 and multiplies by C4, etc. and then sums the value to obtain the total score.

=SUMPRODUCT(B3:B10,C3:C10)

C12 fx =SUMPRODUCT(C4:C11,$B4:$B11)

	A	B	C	D	E	F
1			**Data Rescaled to Common Units (Between 0 and 1)**			
2			**Honda**	**Toyota**	**Ford**	**Nissan**
3	**Measure**	**Weights**	**Civic Hybrid**	**Prius**	**Focus**	**Versa**
4	Purchase price	0.2	0.100	0.050	0.830	0.500
5	Miles per gallon	0.19	0.767	0.867	0.167	0.433
6	Odometer reading	0.12	0.300	0.400	0.100	0.700
7	Body condition	0.15	0.500	0.500	0.500	1.000
8	Air conditioner and heater	0.07	1.000	1.000	1.000	1.000
9	Sound system	0.1	0.750	0.750	1.000	0.750
10	Color	0.02	0.250	0.500	0.000	0.750
11	Body design	0.15	1.000	1.000	0.000	0.500
12	**Total Score**	1	**0.577**	**0.603**	**0.455**	**0.651**

Figure 1.3.2: Use SUMPRODUCT to calculate total score in one step

To obtain Toyota's total score, Judy will use column B but this time multiply it by column D.

=SUMPRODUCT(B3:B10,D3:D10)

Judy recalls that EXCEL allows one to copy and paste functions from one cell into another. She wants to copy the SUMPRODUCT formula from cell C11 to cells D11, E11, and F11. However she notices that for each of the cars, the first range will always refer to B3 to B10. To ensure that column B always appears in the function, she places a $ sign before each B letter. The $ before the B ensures that when the cell is copied into another cell the B column is unchanged. Here is how she wrote the function.

=SUMPRODUCT($B3:$B10,C3:C10)

When Judy copied C11 into D11 the result was

=SUMPRODUCT($B3:$B10,D3:D10).

She repeated this for cells E11 and F11.

Multi-step Method with more Details

The above method determines the total score but does not show the individual measure components of each total score. Thus, Judy is unable to tell how much the purchase price contributes to the total score of each car. She decided to use EXCEL's capabilities to replicate Table 1.3.8 which has the detailed information. (See also Table 1.1.12 and Table 1.2.10.) She set up a new area in the spreadsheet in rows 16 through 26 to calculate the individual subtotals. This is displayed in Figure 1.3.4. To obtain the value in cell C18, she multiplied C3 by $B3. She again placed a $ symbol before the B because she was going to use the copy and paste function to complete the table.

C26 =SUM(C18:C25)

	A	B	C	D	E	F
15			Weight * Common Unit			
16			**Honda**	**Toyota**	**Ford**	**Nissan**
17	**Measure**		**Civic Hybrid**	**Prius**	**Focus**	**Versa**
18	Purchase price		0.020	0.010	0.166	0.100
19	Miles per gallon		0.146	0.165	0.032	0.082
20	Odometer reading		0.036	0.048	0.012	0.084
21	Body condition		0.075	0.075	0.075	0.150
22	Air conditioner and heater		0.070	0.070	0.070	0.070
23	Sound system		0.075	0.075	0.100	0.075
24	Color		0.005	0.010	0.000	0.015
25	Body design		0.150	0.150	0.000	0.075
26	**Total Score**		**0.577**	**0.603**	**0.455**	**0.651**

Figure 1.3.3: Calculate subtotals: multiply weight by common unit and sum the subtotals

Judy then copied cell C18 into cells C19 through C25. Judy then used the SUM function to calculate the total score. In cell C26 she wrote

=SUM(C18:C25)

She then copied the entire column of values C18 through C26 to columns D, E, and F. She now can see the impact of the subtotals on each car's total score. She noticed that the purchase price contributes only 0.02 to Honda's total score of 0.582. In contrast, the purchase price contributes 0.10 to Nissan's total score of 0.636.

Chapter 1 (MCDM) Homework Questions

1. Olivia wants to pursue a career in medicine, but she is not sure which profession would be best for her. After some preliminary research, she narrows her choices to physician, nurse, and pharmacist. Olivia decides to consider four criteria to help structure her decision: professional preparation, personal fulfillment, financial compensation, and lifestyle. The table below shows these criteria and the measures she has decided to use for each.

Criterion	Measure	Type of Scale	Type of Data
Professional Preparation	Schooling		
	Internship		
	Difficulty		
Personal Fulfillment	Job satisfaction		
	Personal interest		
Financial Compensation	Initial salary		
	Median salary		
Lifestyle	Likely schedule		
	Maternity leave		
	Prestige		

   a. Decide which type of scale would be appropriate for each measure, either *continuous-natural* or *categorical-constructed*.

   b. Determine which of the data will have to be collected through *research* and what will be based on personal *opinion*.

   c. The table below shows some of the data Olivia has collected for the professional preparation criterion. Based on the scale ranges, determine what you would consider most preferred and least preferred for each measure.

Criterion	Measure	Scale Range	Physician (M.D.)	Nurse (R.N.)	Pharmacist (Pharm.D.)
Professional Preparation	Schooling (years)	2-8	8	4	6
	Internship (years)	0-4	3	0	1
	Difficulty (rank)	1-3	1	3	2

   d. What else must be done before obtaining common unit values?

   e. Fill in the following table with scores scaled to common units.

Criterion	Measure	Physician (M.D.)	Nurse (R.N.)	Pharmacist (Pharm.D.)
Professional Preparation	Schooling			
	Internship			
	Difficulty			

f. Suppose Olivia weights Schooling at 0.109, Internship at 0.091, and Difficulty at 0.073. Complete the following table with the weighted subtotal score for each measure for each alternative..

Criterion	Measure	Physician (M.D.)	Nurse (R.N.)	Pharmacist (Pharm.D.)
Professional Preparation	Schooling			
	Internship			
	Difficulty			

2. Rana is trying to decide what part time job to take during the school year.

   a. Identify 3 or more criteria she could use to determine her preferred job.

   b. For each criterion, specify at least two measures.

   c. Specify the type of scale for each measure

   d. Assume now the decision involves taking a full-time job in the summer. Identify at least one measure to be eliminated from your list. Identify at least one criterion and measure to be added to the evaluation.

Criterion	Measure

3. Give an example of a measure that could be a continuous scale but you would choose to create a categorical scale instead. Explain your answer.

4. Give an example of a measure that uses a categorical scale, but might not be converted to common units proportionally. Explain your answer.

5. In problem 1.1 of the chapter, Isabelle Nueva is helping her mother and father decide on the best wireless plan for her family.

   a. What additional measures do you think should be considered?

   b. Add and describe a categorical measure for the problem and create 3 categories for this new measure.

   c. Add and describe a numerical measure for the problem.

6. In problem 1.2 of the chapter, Enrique Ramirez is selecting a college to attend.

   a. What additional measures do you think should be considered?

   b. Add and describe a categorical measure for the problem and create 3 categories for this new measure.

   c. Add and describe a numerical measure for the problem.

7. In problem 1.3 of the chapter, Judy is choosing which used car to purchase from among four possibilities.

   a. What additional measures do you think should be considered?

   b. Add and describe a categorical measure for the problem and create 3 categories for this new measure.

   c. Add and describe a numerical measure for the problem.

8. A high school student wants to buy a digital camera. Checking the experts' recommendations, she creates a list of important features and ranks them as follows. She ranks Price as the most important measure and, therefore, assigns 100 points it. Brand name is slightly less important than price. It is ranked 2^{nd} and she assigns 90 points to it. She thinks that having an Anti-shake system is much less important than brand name and assigns 60 points to it. Size of view screen is ranked below anti-shake system and has a little bit less importance, thus she assigned it 55 points. Finally, ease of use is the least important factor with 40 points. Calculate the weight assigned to each measure.

Measure	**Rank**	**Points**	**Weight**
Size of view screen	4	55	
Price	1	100	
Brand name	2	90	
Anti-shake system	3	60	
Easy to use	5	40	
	Total		

9. Suppose you are looking to buy a digital camera for yourself.

   a. Suggest and add a relevant categorical measure in the table below. Describe the new measure.

   b. Suggest and add a relevant numerical measure in the table below. Describe the new measure.

   c. Use your personal preferences and rank the measures. Then, assign points to each measure and calculate the weight of each measure.

	Assign		Calculate
Measure	**Rank**	**Points**	**Weight**
Size of view screen			
Price			
Brand name			
Anti-shake system			
Easy to use			
New categorical measure:			
New numerical measure:			
	Total		

10. Kim is interested in purchasing a desktop computer for her office. After reviewing the specification of different models, she ended up with the following measures. Classify each measure as numerical or categorical.

Measure	Type: Numerical or Categorical
Computational power	
Monitor size	
Years of warranty	
Operating system	
Price	

11. A high school has selected one of its students to be the chair of a committee planning a class trip. One of her first responsibilities is to pick a co-chair for planning the trip. Suggest two measures for each criterion that could be used to help select a co-chair for planning the trip. In specifying measures be sure they are relevant to this co-chair selection. Specify the type of each measure.

Criterion	Measure	Type: Numerical or Categorical
Knowledge		
Reliability		
Personality		

12. Sam and his wife were just married and are looking for an apartment in a safe area close to Sam's school. After discussing their preferences, they came up with the following measures that are very important to them.

Measure	Description
Spaciousness	Size and design
Price	Monthly rental
Condition	Freshly painted, floors, age of appliances
Apartment building rating	Based on previous tenants' rating in www.apartmentrating.com

a. After searching in a 10-mile radius around his school, they ended up with the following three apartments they like. Sam summarized the data as follows:

Measure	Ap1	Ap2	Ap3
Spaciousness	Good	Medium	Poor
Price ($/month)	700	650	550
Condition (0-1)	0.6	0.9	0.7
Apartment building rating (between 1 and 5)	4	4.5	3.8

b. Specify the range for each measure and then determine the common unit for each of them. Insert the common units in the following table.

Measure	Ap1	Ap2	Ap3
Spaciousness			
Price			
Condition(0-1)			
Apartment building rating			

c. After considering the measures, Sam and his wife ranked the measures as in the following table. Use the assigned points to calculate the weights.

Measure	Rank	Points	Weight
Spaciousness	3	70	
Price	1	100	
Condition(0-1)	2	90	
Apartment Building Rating	4	50	

d. For each alternative, calculate the product of the weight and the corresponding common unit for each measure. Determine the total score for each alternative.

Measure	Ap1	Ap2	Ap3
Spaciousness			
Price			
Condition			
Apartment Building Rating			
Total Score			

e. Which alternative is ranked 1st and what measures contribute the most to it being ranked 1st?

13. James and George are seeking a team member for their final project in their senior year that will involve a lot of data analysis. It is a very demanding project that requires a wide range of skills. To help evaluate potential teammates, they created the following list of measures.

Measure
Writing Skills
GPA of Math courses
Total GPA
Reliability and commitment
Communication skills

a. After considering all their classmates who were not yet assigned to any project, they ended up with following three people. They summarized the data for these three as follows:

Measure	Ed	Ken	Thad
Writing skills	Excellent	Acceptable	Good
GPA in math courses	3.5	3.9	3.0
Total GPA	3.6	3.8	3.3
Reliability and commitment	Acceptable	Good	Good
Communication skills	Good	Acceptable	Excellent

b. Specify the range for each measure and then determine the common unit for each of them. Insert the common units in the following table. (Assume proportionality.)

Measure	Ed	Ken	Thad
Writing skills			
GPA in math courses			
Total GPA			
Reliability and commitment			
Communication skills			

c. They are not sure how to rank the measures. Based on your personal preferences, rank the measures and fill out the rest of table.

Measure	Rank	Points	Weight
Writing skills			
GPA in math courses			
Total GPA			
Reliability and commitment			
Communication skills			
	Total		

d. For each alternative, calculate the product of the weight and the corresponding common unit for each measure. Determine total score for each alternative.

Measure	Ed	Ken	Thad
Writing skills			
GPA in math courses			
Total GPA			
Reliability and commitment			
Communication skills			
Total Score			

e. Which alternative is ranked 1st and what measures contribute the most to him being ranked 1st?

14. Neil is trying to find a location in Michigan to open a convenience store. Location is very important for convenience stores. Thus, he wants to be very precise in this process. After talking to some consultants and other store managers, he plans to use the following measures.

Measure	Description
Traffic through intersection	Daily number of the cars passing the intersection
Population within 2 mile	Total population over the age of 15
Distance to the nearest competitor	Miles to nearest convenience store
Cost of the property	Purchase price of property

a. After considering all available properties in the area, he ends up with the following three locations. The data for these three locations is summarized below.

Measure	L1	L2	L3
Traffic through intersection (vehicles)	16,000	15,000	19,000
Population within 2 miles	50,000	45,000	55,000
Distance to the nearest competitor (miles)	1.5	2	0.5
Cost of the property ($)	210,000	180,000	250,000

b. Specify the range for each measure and then determine the common unit for each of them. Insert the common units in the following table.

Measure	L1	L2	L3
Traffic through intersection			
Population within 2 mile			
Distance to the nearest competitor			
Cost of the property			

c. After considering the measures, he ranks the measures as in the following table. Use assigned points to calculate the weights.

Measure	Rank	Point	Weight
Traffic through intersection	2	85	
Population within 2 mile	3	80	
Distance to the nearest competitor	4	70	
Cost of the property	1	100	
Total			

d. For each alternative, calculate the product of the weight and the corresponding common unit for each measure. Determine total score for each alternative.

Measure	L1	L2	L3
Traffic through intersection			
Population within 2 mile			
Distance to the nearest competitor			
Cost of the property			
Total Score			

e. Which alternative is ranked 1st and what measures contribute the most to it being ranked 1st?

15. Gerald and his friends are trying to decide where to go for spring break.

a. Identify 3 or more criteria he and his friends could use to determine the preferred spring break location..

b. For each criterion, specify at least two measures.

c. Specify the type of scale for each measure

16. Identify a multi-criterion decision context that you or anyone in your family is facing within the next year. Explain how this is a multi-criterion decision.

Chapter 1 Summary

What have we learned?

We have learned that the multi-criteria decision making process provides a framework for making a subjective decision when considering several alternatives, each of which has advantages and disadvantages. As the person making the decision, you must structure the decision. What criteria or objectives will be considered? What measures of your criteria will be included? How will you rank and weight these measures to help make a decision that is best for your values and priorities?

This process allows for direct comparison and evaluation of complex alternatives. The steps are as follows:

1. Identify Criteria and Measures
2. Collect Data
3. Find the Range of Each Measure
4. Rescale Each Measure to a Common Unit
5. Conduct an Interview to Calculate Weights
6. Calculate Total Scores
7. Interpret Results

Terms

Categorical Measure	A measure whose scores are classifications
Common Unit	A value that varies from 0 to 1, where 0 always represents the worst value, 1 the best value, and intermediate values are found using a proportional or non-proportional scale
Continuous Measure	A measure whose scores are numeric values that can take on any value in a certain range
Categorical Measure	A measure that is divided into distinct categories. These categories could be natural such as the color of a car or they can be created by grouping numerical values into ranges.
Criteria	Objectives or aspects of the alternatives that you wish to either maximize or minimize
Measure	A trait that will quantify an aspect of a criterion
Proportional Scale	The rescaled score for intermediate values of continuous measures (calculated by dividing the difference between the particular score and the least preferred score by the scale range). A categorical measure can also use a proportional to assign a common unit to an intermediate category.
Non-Proportional Scale For Categorical Measure	The rescaled score for intermediate values of categorical measures need not be proportional to where the category falls on the list. For example, with 3 categories, the intermediate category need not be assigned a common unit of 0.5. (Non-proportional scales can also be used for continuous variables as well. This concept is beyond the scope of this course.)
Scale Range	The range of possible values for a measure.
Total score	For each alternative, multiply the rescaled score by the weight for each measure. The sum of all these weighted, rescaled scores is the total score.
Weighted Subtotal Scores	The rescaled score for each measure, weighted according to its importance (calculated by multiplying each scaled score by the corresponding weight of the measure)

Chapter 1 (MCDM) Objectives

You should be able to:

- List the sequence of steps in the multi-criteria decision making process
- Explain the purpose of each step in the process
- Identify criteria you will use to choose between several alternatives
- Select measure(s) for each criterion
- Distinguish between categorical and continuous measures
- Determine scale types and ranges for measures
- Scale scores
- Rescale scores to common units
- Weight scores for each measure
- Calculate a total score for each alternative
- Evaluate the results of the multi-criteria decision making process by comparing the strengths and weaknesses of the top two alternatives

Chapter 1 Study Guide

1. Explain why the Multi-Criteria Decision Making (MCDM) process is useful.

2. Discuss the differences between a *criterion* and a *measure*.

3. When choosing between the same alternatives, why might you and a classmate, both using MCDM, come to a different decision?

4. Compare and contrast *continuous* and *categorical* measures.

5. Give an example of a scale range in which one end is most preferable for you, but the other end may be preferable to a classmate. Explain.

6. Why do we scale all scores between zero and one?

7. Describe how scores are scaled differently for continuous and categorical measures.

8. Describe how scaled scores are rescaled to common units differently for continuous and categorical measures.

9. Identify which steps in MCDM involve you inserting your own preferences and priorities into the process and describe how this occurs?

10. What role do the weights of the measures play in determining which alternative is the best?

11. Describe the process that occurs from collecting raw data for measures to obtaining a total score for an alternative.

12. Should you always choose the alternative with the highest total score?

References

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