FROM RATES TO ALGEBRA

Authentic Decision Scenarios

Homework

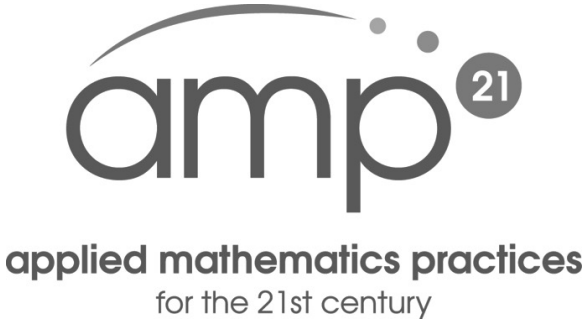
Basketball

Arcade Games

Car Speed & Texting

Tipping Water Buckets

State Min Wage



FROM RATES TO ALGEBRA

Using Authentic Problem Contexts

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Special Acknowledgements:
Graphics: Lindsey Potoff

Applied Mathematics Practices for the 21st Century (AMP21) is an affiliation of professors and mathematics educators who share a common desire to bring relevance to K-12 mathematics by using authentic problem contexts in teaching and developing mathematics concepts and skills. The lead faculty are Kenneth Chelst (College of Engineering) and Thomas Edwards (College of Education) who are both professors at Wayne State University in Detroit, MI.

AMP21 is a Non-Profit developer and provider of curriculum that is aligned with the eight Standards for Mathematical Practice in the Common Core State Standards. Our team has published two textbooks for high school 1) Algebra and Mathematical Modeling and 2) Probability and Mathematical Modeling. Our new curriculum development focuses on middle school topics central to proportional reasoning: percentages, rates, ratios and proportions. Everything is presented in authentic problem contexts that involve making decisions. AMP21 offers professional development workshops in conjunction with local universities to help teachers and schools develop the programs needed to enable students to succeed in the global economy. In addition, this curriculum can form a basis for project based learning in mathematics.

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Dear Teacher,

As mathematics teachers, we are all challenged to provide our students with complex problems to solve. Our wish for their experience in mathematics is that they understand that real world scenarios involving mathematics often take more than one minute or one statement to solve. Where do we find those materials? Mathematics is used every day in business, industry, marketing, etc. to make decisions, plan ahead, and predict outcomes. Many textbook examples do not engage students or help them see the value of mathematics.

This book is our attempt to provide you with authentic, meaningful, engaging scenarios that showcase the importance of mathematics in the world. As state and national assessments (PSAT/SAT) are beginning to test more complex mathematics problems through the contexts of science and social studies, we find ourselves looking for practice to develop those critical thinking skills necessary for achievement.

According to the College Board website, the revised PSAT/SAT Mathematics Test "focuses in-depth on three essential areas of math: Problem Solving and Data Analysis, Heart of Algebra, and Passport to Advanced Math. Problem Solving and Data Analysis is about being quantitatively literate. It includes using ratios, percentages, and proportional reasoning to solve problems in science, social science, and career contexts. The Heart of Algebra focuses on the mastery of linear equations and systems, which helps students develop key powers of abstraction. Throughout the PSAT/SAT, you'll be asked questions grounded in the real world, directly related to work performed in college and career." This book focuses on using rates to solve problems in business, industry, marketing, government, sports, and management. The Mathematics section on the PSAT/SAT "features multi-step applications to solve problems in science, social science, career scenarios, and other real-life situations. The test sets up a scenario and asks several questions that give you the opportunity to dig in and model it mathematically."

The scenarios in this book provide students with similar experiences. Students are given an authentic, meaningful scenario, asked to answer questions, dig deeper and model the situation mathematically. Some activities involve inter-

preting/analyzing data from a graph or table, while others involve creating a graph or table with given data or data they have collected themselves. Activities engage students in making transitions from rates to algebraic expressions to solving equations.

This book provides a teacher guide for each of the activities. Each lesson begins with a brief Number Talk suggestion that directly relates to mindful mental mathematics for that particular lesson. The guide utilizes the lesson-planning framework of Thinking Through a Lesson Protocol which was developed through the collaborative efforts (lead by Margaret Smith, Victoria Bill, and Elizabeth Hughes) of the mathematics team at the Institute for Learning and faculty and students in the School of Education at the University of Pittsburgh. The lessons are structured in the Launch, Explore, Summary format. Suggestions have been provided in some of the columns. However, the "Who - Selecting/Sequencing" column (in the middle) is intentionally left blank for the teacher to make notes during the Explore portion of the lesson. It is intended for the teacher to sequence which groups/strategies will be presented and in what order during the Summary (most important) portion of the lesson. Much as a skillful conductor leads an orchestra, mathematics teachers must lead their students to a deeper understanding of the mathematics. This is done through collaborative discussions involving analyzing, critiquing, justifying, and making the mathematical connections. Each activity is expected to take between one and two class periods depending on your students' prior knowledge and the length of the class periods. Estimated time suggestions are provided, but can be easily adjusted to fit yours and your students' needs.

Enjoy!

Kenneth Chelst, Thomas Edwards, Debbie Ferry, Marianne Srock

Introduction to Rates

Rates are a common descriptor of performance in diverse settings. They are readily used to compare performances across geographies (US States), time (2019 vs 2020), and different products (Honda vs Ford). The concept of rate is a fundamental element of algebraic expressions. The numerical coefficient before the variable is often a rate. For example, if t represented the variable time in hours, then $60t$ represents the distance traveled in time t at a speed of 60 miles per hour.[1]

A rate is an example of ratio in which the numerator and denominator can be different units. Travel speed is a good example of non-commensurate values. The numerator is a measure of distance and the denominator is time the result, miles per hour. Other examples of different units of measurement include: miles and gallons, dollars and pounds, deaths and miles driven.

The denominator may be a single unit and the rate is per unit, as in miles per gallon. Alternatively, the denominator may be a standard number. For example, automobile fatality rates are often reported per 100 million vehicle miles driven. The standard denominator facilitates comparisons across geography. In 2020 the rate for the U.S. was reported as 1.37 fatalities per 100 million vehicle miles traveled (VMT). The rates by state ranged from a low of 0.63 for Massachusetts to a high of 1.97 fatalities for South Carolina. A standard denominator allows comparisons across years (2019 vs 2020) as driving behavior might change. The total miles driven was affected by the spread of Covid 19. Total VMT for 2020 decreased by 11 percent from 2019, from 3,262 billion to 2,904 billion.

1 Many of the concepts developed in this introduction are discussed in a journal article. Ozgün-Koca, S., Edwards, T. G., and Chelst, K. R. (2013) "Exercise Away the Big Mac: Ratios, Rates, and Proportions in Context," *Mathematics Teaching in the Middle School*, 19(3), 184-188

The standard number used in the denominator is usually chosen to make the rate more meaningful. Imagine using a different denominator for fatality rates, for example, one million miles. The 2020 calculated rate would then be 0.0137 fatalities per million miles travelled. Most people find it easier to comprehend 1.37 fatalities than 0.0137 fatalities.

Sometimes, measures of rates vary when applied to the same process in dissimilar contexts. For example, the speed of an automobile is usually given in miles or kilometers per hour. Doing so facilitates calculating or estimating the time needed to travel a known distance. However, when thinking about stopping time, or reaction time, when driving an automobile, feet per second, or meters per second are more meaningful. It is much easier to visualize how far you would travel, while reacting to a need to stop, if you think of your speed as 88 feet per second rather than 60 miles per hour, although these two rates are equivalent.

In some examples, there is an option to exchange the numerator and denominator to redefine the rate. In the U.S., miles per gallon (mpg) is the standard measure of fuel economy. It enables a driver to forecast how far the car can travel before needing to refuel. However, improved mpg does not directly translate into cost savings. Consider the savings when fuel economy improves from 25 mpg to 30 mpg. This a 20% improvement. Envision the multi-step calculation of dollar savings for a driver who travels 1,000 miles per month. In contrast, most countries report fuel economy as gallons per 100 miles or liters per 100 kilometers.

In the examples cited above, the units of measure for the numerator (miles) and denominator (gallons) are different. Sometimes the two units of measurement are commensurate but refer to different populations. The number of people in prison in the US and the US population o are essentially the same units: human beings. The rate of prison incarceration in the U.S. is calculated by dividing the number of people in prison by the total population. The U.S. rate is among the highest in the world with 784 people per 100,000 of population. These rates are often calculated for different subgroups. For example, if the numerator and denominator were limited to adults, the US rate would be 962 adults per 100,000 adults.

The 15 scenarios in the book offer numerous opportunities that teachers can use to lead a discussion of the meaning of specific measures. In standard textbooks measures are simply given and all the student does is calculate a rate or apply it to a short word problem. We believe, our text provides teachers many meaningful applications that can be used to lead students to a deeper understanding of mathematics.

Have fun!

Activity Title	Context	Mathematical Goals
Activity 1: Germaine's HW	Managing time commitment for HW.	• Read a scenario and use rates to solve multi-step problems. • Understand there are two unit rates associated with each ratio. • Calculate rates. • Compare rates. • Work with rates in a meaningful context familiar to students.
Activity 2: Which Rate? Sixth-Man in Basketball	Deciding which of two different rates is the better measure of a 6^{th}-man's value to the team.	• Read a scenario and use rates to solve multi-step problems. • Understand there are two unit rates associated with each ratio. • Calculate rates. • Compare rates. • Work with rates in meaningful contexts.

Activity Title	Context	Mathematical Goals
Activity 3: Fuel Economy	Deciding which rate to use, miles per gallon or gallons per 100 miles.	• Read a scenario and use rates to solve multi-step problems. • Understand there are two unit rates associated with each ratio. • Transition into the use of algebra to answer a question. • Evaluate an algebraic expression for different values of the variable. • Solve an algebraic equation • Work with rates in meaningful contexts
Activity 4: Jasmine's Lemonade Stand	Determining the most effective strategy to earn a profit.	• Read a scenario and use rates to solve multi-step problems. • Understand unit rates and use them to solve problems. • Transition into the use of Algebra to answer a question. • Evaluate an algebraic expression for different values of the variable. • Solve an algebraic equation. • Work with rates in a meaningful context familiar to students.

Activity Title	Context	Mathematical Goals
Activity 5: Arcade Games	Finding an efficient way to win enough tickets playing arcade games to earn a specific prize.	• Read a scenario and use rates to solve multi-step problems. • Understand there are two unit rates associated with each ratio. • Transition into the use of algebra to answer a question. • Evaluate an algebraic expression for different values of the variable. • Solve an algebraic equation. • Work with rates in a meaningful context familiar to students.
Activity 6: Car Speed and Texting	Deciding which rate to use, miles per hour or feet per second when thinking about driving safety.	• Read a scenario and use rates to solve multi-step problems. • Understand there are two unit rates associated with each ratio. • Transition into the use of algebra to answer a question. • Evaluate an algebraic expression for different values of the variable. • Solve an algebraic equation. • Work with rates in meaningful contexts.

Activity Title	Context	Mathematical Goals
Activity 7: Lawn Service	Finding efficient work strategies.	• Read a scenario and use rates to solve multi-step problems. • Understand individual work rates and combined work rates. • Transition into the use of Algebra to answer a question. • Evaluate an algebraic expression for different values of the variable. • Solve an algebraic equation. • Work with rates in a meaningful context familiar to students.
Activity 8: Exercise and Calories	Determining effective strategies to lose weight and maintain the loss.	• Read a scenario and use rates to solve multi-step problems. • Understand there are two unit rates associated with each ratio. • Transition into the use of Algebra to answer a question. • Evaluate an algebraic expression for different values of the variable. • Solve an algebraic equation. • Use and analyze data from a table. • Work with rates in a meaningful context familiar to students.

Activity Title	Context	Mathematical Goals
Activity 9: Tipping Buckets	Using flow rates to efficiently fill buckets and tanks.	• Read a scenario and use rates to solve multi-step problems. • Transition into the use of Algebra to answer a question. • Evaluate an algebraic expression for different values of the variable. • Solve an algebraic equation. • Work with rates in a meaningful context familiar to students.
Activity 10: Franchise Wages	Determining the revenue necessary to cover the cost of workers' wages at a fast-food franchise.	• Read a scenario and use rates to solve multi-step problems. • Understand there are two unit rates associated with each ratio. • Transition into the use of algebra to answer a question. • Evaluate an algebraic expression for different values of the variable. • Solve an algebraic equation. • Work with rates in meaningful contexts.

Activity Title	Context	Mathematical Goals
Activity 11: Pools R Clean, Inc.	Efficiently managing a pool-cleaning business	• Read a scenario and use rates to solve multi-step problems. • Understand there are two unit rates associated each ratio. • Calculate rates. • Compare rates. • Write an algebraic expression and solve algebraic equations. • Complete a table. • Work with rates in a meaningful context.
Activity 12: Teacher to Student Ratios	Determining staffing requirements at two schools	• Read a scenario and use ratios/rates to solve problems. • Solve an algebraic equation. • Read /interpret information from a table. • Work with ratios/rates in meaningful contexts. • Round decimals appropriately depending on context.

Activity Title	Context	Mathematical Goals
Activity 13: Light Bulbs	Determining annual energy savings for various types of light bulbs.	• Read a scenario and use rates to solve multi-step problems. • Calculate electrical energy savings based on cost per kilowatt hour. • Transition into the use of algebra to answer a question. • Evaluate an algebraic expression for different values of the variable. • Solve an algebraic equation. • Work with rates in meaningful contexts.
Activity 14: Relay for Life Walkathon	Comparing different strategies for raising donations from a walk-a-thon.	• Read a scenario and use rates to solve multi-step problems. • Calculate rates to determine monetary goals. • Compare rates in algebraic equations and graph. • Work with rates in a meaningful context familiar to students. • Read and interpret line graphs. • Plot points on line graphs. • Interpret points on a graph in terms of the context of the problem.

Activity Title	Context	Mathematical Goals
Activity 15: State Minimum Wage	Investigating minimum wage rates to determine their effect on annual income before and after deductions.	• Read a scenario and use rates to solve multi-step problems. • Transition into the use of algebra to answer a question. • Evaluate an algebraic expression for different values of the variable. • Solve an algebraic equation. • Calculate a percent of a number and its percentage complement. • Find percent increase. • Work with rates in a meaningful context familiar to students.

The syntactic link between percentages and rates is via the Latin roots of percent, *per* and *centum*. In numerical contexts, the *per* can be interpreted "for each", while the *centum* translates directly as "100". Thus, 50% can be rendered "50 for each 100." Then when we use "per" to express a rate, which we usually do, 60 miles *per* hour can be rendered as "60 miles *for each* hour."

Summary of Activity

There is a total of 15 activities. Each activity is designed to explore a different application of a rate. The activities are all embedded in authentic real-world scenarios that involve making a decision.

1. **Germaine's Homework: Time Commitment** - In this activity, a student, Germaine, has a lot of homework that he must complete over the weekend. He has two reading assignments, finishing a novel

for his English class and two chapters from his history textbook. He also has 20 mathematics word problems to solve. Using two different reading rates and his rate of solving math problems, Germaine's older sister, Charlene, helps him develop a plan to better manage his time in order to complete all of the assignments. The activity ends with a number of practice problems set in similar contexts.

2. **The Sixth Man in Basketball: Which Measure to Use?** After carefully explaining the term "sixth man," the second activity involves a scenario in which an NBA team must decide which of two "free agents" to try to sign in order to fill the role of their sixth man. In doing so, two different scoring rates are considered. Three practice problems are provided. They involve a number of different rates associated with basketball, and are set in the context of the WNBA.

3. **Fuel Economy: Which Measure to Use?** In this activity, students explore the relationship between the fuel economy (miles per gallon), and the amount spent for gasoline. Along the way, they will learn that a 25% increase in fuel economy does not necessarily translate into a 25% savings in the cost of the fuel used. They will also explore the question of whether gallons per 100 miles driven is a better measure of fuel economy than miles per gallon.

4. **Jasmine's Lemonade Stand: Earning Money for a Video Game** - In the fourth activity, a middle school student, Jasmine, runs a very successful lemonade stand in the summer. This activity involves Jasmine's use of a number of rates, with her mother's help, to more efficiently operate her lemonade business. They will also transition into the use of algebra to assist in answering a question of interest. Seven practice problems that extend the lemonade stand activity close out the activity.

5. **Arcade Games: Efficient Way to Earn Tickets** - In activity 5, students use rates to explore the potential cost of paying arcade games to win tickets with which a bracelet or bracelets will be purchased. They will also transition into the use of algebra to assist in answering a question of interest.

6. **Car Speed & Texting: Which Measure to Use?** In this activity, students consider one of the dangers of texting while driving by exploring

stopping distance at various speeds. Along the way, they must decide on the best rate for measuring the speed of an automobile for this purpose, miles per hour or feet per second.

7. **Lawn Service: How Many Customers?** In the seventh activity, two college students decide to set up a lawn service in order to earn money for school. They have a goal to earn at least $1.000 each, and in the activity, they determine they best way to take advantage of their two different work rates. In order to do that, they must figure out what their work rate would be if they work together on each job rather than working individually on separate jobs.

8. **Exercise Away Calories: Walking, Biking, or Swimming –** In the eighth activity, a newly promoted assembly line supervisor is concerned about his new job being less physically demanding. With the help of his two children, he explores various physical activities (e.g., walking, bicycling, running, swimming) that he could do in his spare time. The complicating factor is that he only has a half-hour of free time available each day for physical exercise.

9. **Tipping Water Buckets: How Long Does it Take to Refill? –** In Activity 9, two workers at a water park must plan the recycle time for tipping buckets of four different sizes. To do this, they use water flow rates of various different pipes as well as set up and solve algebraic equations in order to achieve a desired recycle time.

10. **Franchise Wages and Annual Cost –** In this activity, two owners of a fast-food franchise use pay rates to determine the annual cost of the staffing levels they are planning. Then they compare that cost to their expected annual revenue.

11. **Pools R Clean, Inc.: Earning & Helping Family Business –** This activity is about a family business in which a woman cleans swimming pools weekly. She enlists the aid of her daughter, and they consider two different rates for measuring their efficiency. The activity concludes with the daughter using her rate of pay to determine how many hours she would need to work to earn enough money to pay for a summer camp.

12. **Teacher-Student Ratios: How Many Teachers are Needed? –** In this activity, a school district administrator and a teacher plan the number of teachers at grades K-3 in two different schools that will be needed for the coming year. They also consider the possible need to transfer some students from one school to the other in order to meet state teacher-to-student quotas. The activity ends with a similar consideration for hiring math tutors in each of the schools.

13. **Light Bulbs: Using Less Energy –** In activity 13, a middle school student considers replacing the incandescent light bulbs in her bedroom with new LED bulbs. In order to make an informed decision, she decides to calculate the amount and the cost of the electrical energy used by the incandescent bulbs. Then she repeats those calculations for the LED bulbs with which she is contemplating replacing the incandescent bulbs currently in use.

14. **Relay for Life Walkathon: Raising Money –** In this activity, three middle school students are going to walk laps on the school's quarter-mile track in order to earn money for charity through a Walkathon. They use rates to compare and contrast their three different strategies for soliciting donation pledges from potential donors.

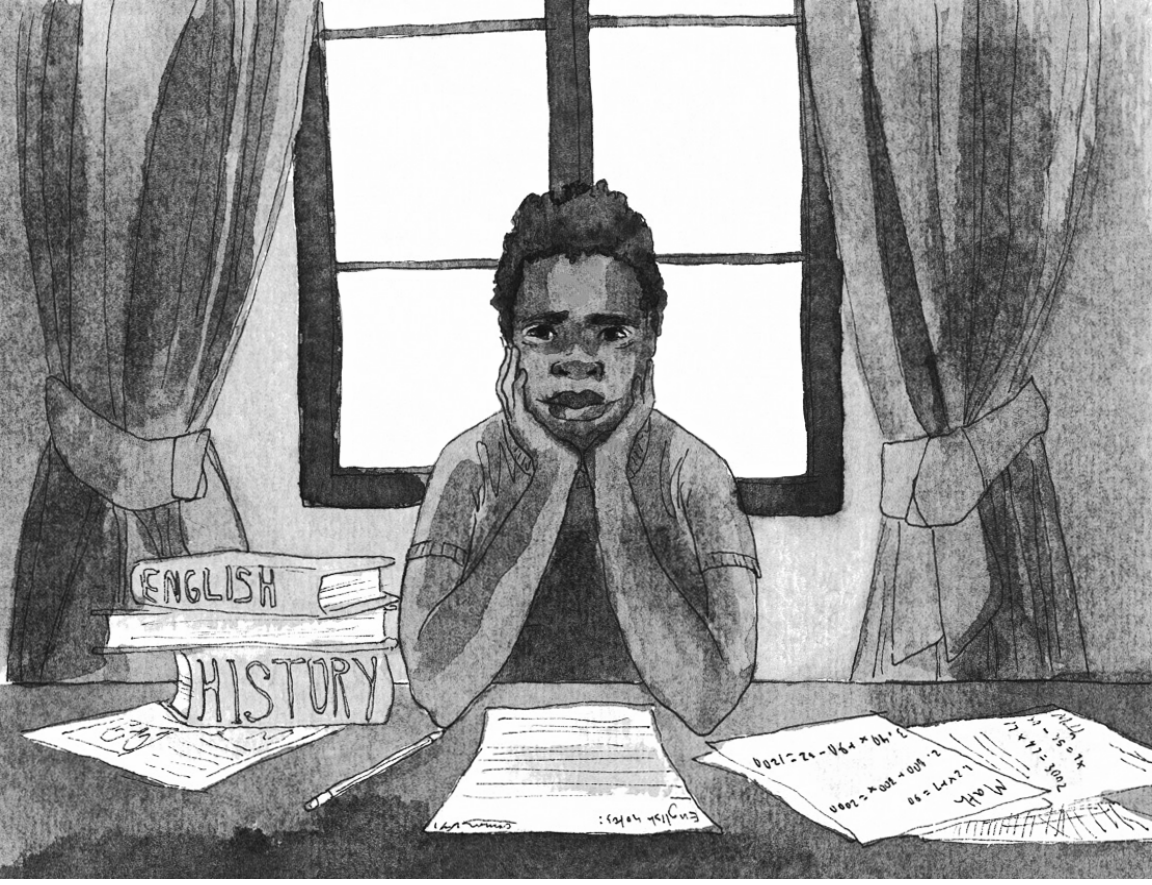
15. **State Minimum Wage: Minimum Wage and Annual Income –** Our last activity concerns two leaders in an effort to raise their state's minimum wage. Currently, the minimum wage in their state is the same as the Federal minimum wage, $7.50/hour. They are working on a ballot proposal. In the activity, they use the Social Security and Medicare tax rates to help them decide what new minimum wage rates to propose for their state.

All 15 activities use rates to make decisions in a meaningful context. In designing these activities, we strove to include the use of algebra in a natural way. Thirteen of the activities involve the use of algebraic expressions or equations. Most of the activities make use of tables of data and a few make use of graphs. At the end of each activity, we present a simple project idea related to the context of the activity. Twelve of the projects involve students collecting and analyzing data.

Representations Used in Rates Activities

	Activity	Concepts	Table	Graph/ Chart	Algebra	Project Idea
1	Germaine's Homework	Unit rates associated with a ratio, Calculate rates, Compare rates.	N	N	N	Y
2	The Sixth Man	Unit rates associated with a ratio, Calculate rates, Compare rates	N	N	N	Y
3	Fuel Economy	Unit rates, Calculate rates, Compare rates.	N	N	N	Y
4	Lemonade Stand	Unit rates, Algebraic expressions and equations.	Y	N	Y	Y
5	Arcade Games	Unit rates, Algebraic expressions and equations.	Y	N	Y	Y
6	Speed of Car	Unit rates, Algebraic expressions and equations.	Y	N	Y	Y
7	Lawn Service	Unit rates, Combine two unit rates of the same type, Algebraic expressions and equations.	Y	N	Y	Y
8	Exercise & Calories	Unit rates, Analyze & use data, Algebraic expressions and equations.	Y	N	Y	Y
9	Tipping Buckets	Unit rates, Algebraic expressions and equations.	Y	N	Y	Y

	Activity	Concepts	Table	Graph/ Chart	Algebra	Project Idea
10	Franchise Wages	Unit rates, Algebraic expressions and equations.	Y	N	Y	Y
11	Pools R Clean, Inc.	Calculate and compare rates, Algebraic expressions & equations	Y	N	Y	Y
12	Teacher/ Student Ratios	Ratios & associated rates, Algebraic equations, Read & interpret tables, Round decimals appropriately	Y	N	Y	Y
13	Light Bulbs	Unit rates, Algebraic expressions and equations.	Y	Y	Y	Y
14	Relay-4-Life	Calculate rates, compare rates in algebraic equations and graphs, Read and interpret line graphs, Plot points on line graphs.	Y	Y	Y	Y
15	State Minimum Wage	Percent of a number, Complementary percentage, Percent increase, Algebraic expressions & equations	Y	Y	Y	Y



Activity 1: Germaine's Homework

Time Commitment

Mathematical Goals

The student will use rates to determine the amount of time it takes to read a novel, informational text, and solve math problems.

The student will:

- Read a scenario and use rates to solve multi-step problems
- Understand there are two unit rates associated with each ratio
- Calculate rates
- Compare rates
- Work with rates in a meaningful context familiar to students

Before the lesson, practice mental mathematics (5-10 minutes).

Number Talk Possibilities: Select two or three depending on student abilities.

State reading 20 pages in 5 minutes as three different ratios

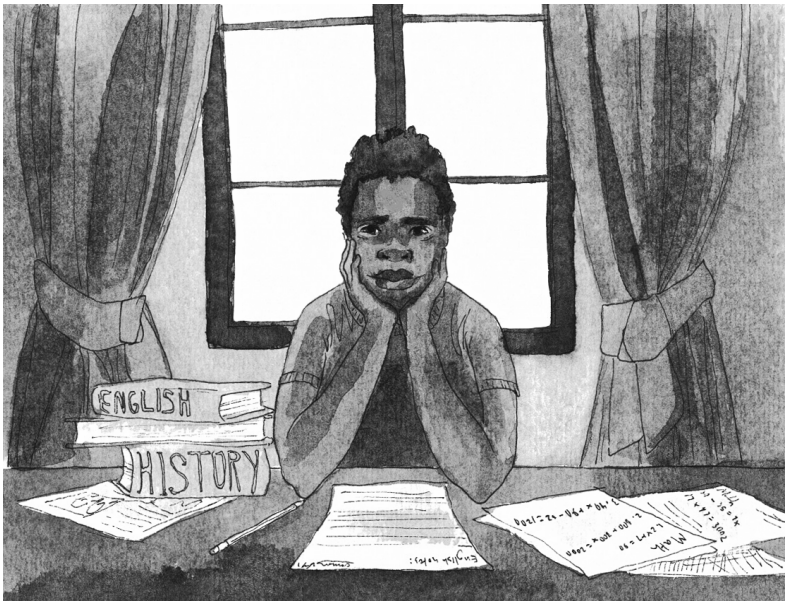
State a reading rate of 20 pages in 5 minutes two different ways

State a unit rate of reading 20 pages in 5 minutes two different ways

- 20 pages in 5 minutes is the same as ___ pages in 1 minute
- 20 pages in 5 minutes is the same as 1 page in __ minutes

Germaine's Homework

Germaine Studious has a lot of school work to complete over the next few days. He has 155 pages in a novel left to read for English class. He must solve 20 word problems for mathematics. Germaine also needs to read two chapters in an American history textbook to prepare for a quiz next week. There are 18 pages in each chapter. Currently, he is planning to spend seven hours over the weekend to complete the work but he is not sure this will be enough time.



"Charlene," he asked his older sister, "there's no way I can finish all of this work. I don't know how you manage!"

"*Manage*," she said, "is the operative word." "You work hard, but you don't work smart. What you need to do is budget your time. With a little planning, you'll be surprised at how much you can accomplish. Let's start by timing how long it takes you to read the next five pages of your novel."

Germaine took her advice. It took him 10 minutes to read five pages. Often, when talking about rate, the denominator is time. He determined his reading rate, R,

$$R = 5 \text{ pages}/10 \text{ minutes} = 0.5 \text{ pages per minute.}$$

"OK, he said. If I divide the number of pages by the number of minutes it took me to read them, I get a rate of .5 pages a minute. Great! Now how am I supposed to figure how long it will take me to do another 150 pages? I hate decimals!"

"Okay," his sister told him. "How about if you try to figure out how long it took you to read each page. I promise there won't be any decimals!"

$$10 \text{ minutes} / 5 \text{ pages} = 2 \text{ minutes per page.}$$

With this number, it was easy to determine the time it would take him to finish the novel.

$$\text{Time} = 2 \text{ minutes per page} \times 150 \text{ pages} = 300 \text{ minutes.}$$

Germaine was surprised that he would need 5 hours just to finish the novel. He felt if he focused he could read faster. He tried again and read the next five pages in only 8 minutes.

1. What is Germaine's reading rate when he focuses?

2. How long will it take him to read the remaining 145 pages with the faster reading rate?

Germaine then proceeded to solve the first pair of word problems in his mathematics text. It took him 14 minutes to solve the two problems.

3. How long will it take Germaine to solve the remaining 18 problems?

"Great!" Germaine said. At this rate, I should be able to finish the math pretty quickly. It looks like the problems get easier with practice. After all, the second problem took only six minutes to complete."

4. "How long will it take Germaine to solve the remaining 18 problems if, he used the second problem to estimate his rate of solving word problems?"

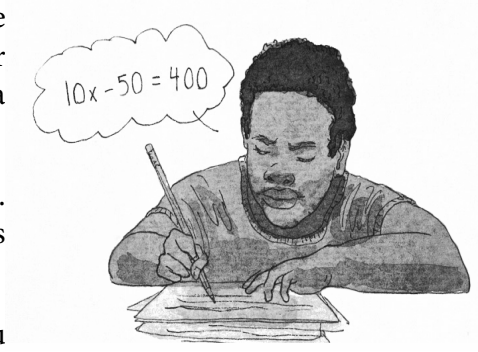
"Not so fast," Charlene said. "That hasn't been my experience with math. Take a few minutes to look over the whole set of problems. You're likely to find out that the problems at the end are *more* challenging – not less. You'll want to allow for more time to do those."

"You're right. Of the 18 problems, those last five have like a million steps. I'll need to figure twice as much time for each problem as it took me to solve the second word problem," Germaine told her.

5. How long will it take Germaine to solve these five problems?

6. How much time should Germaine plan for completing the math homework?

Germaine still needed to set aside time to read the history book and prepare for the quiz. The two chapters contained a total of 36 pages.



"The history should be a breeze," he said. "I already know that my reading rate is two minutes per page."

"Not so fast," his sister told him. "You may want to do another test run and recalculate."

"Spoilsport!" Germaine said, but he took her advice.

7. Why do you think the reading rate he found earlier should not be used in planning his American History studying?

Germaine read just two pages. It took him six minutes to read these two pages.

8. How long will it take Germaine to read the remaining 34 pages?

"That was really good," Charlene said to Germaine. "But I think you're forgetting something."

"Come on," Germaine said. "I read the pages and I timed myself. What's to forget?"

"Well," Charlene said, "You're not reading this for pleasure."

"I'll say."

"You're going to be tested on this material. Don't you think you should be taking…?"

"A break?" Germaine said hopefully. "Never mind, I was just kidding. You're right, Charlene. I need to allow for time to take notes while I read."

Germaine went back and took notes on the first two pages. It took him an additional five minutes.

9. How much time does Germaine need to add to his total for taking notes? Now what is the **total** time required to study for American history?

Germaine and Charlene's dad, Frank Lee Studious, works for Amazon.

"What were the two of you saying about note taking?" he asked.

"Dad, it's impossible. "At this rate I'll be in middle school the rest of my life," Germaine said.

"Let me help you," Mr. Studious said. "You're a smart boy and a hard worker. But working hard is often not as effective as working smart."

"That's what I said," Charlene exclaimed. Germaine said nothing.

"Here," their father said. "Let me show you how you can get by with a little help from your electronic friends. There is new technology available on electronic books that will enable you to take notes much more efficiently. Let me show you."

Germaine tried it out. "Hey! You're right," he told his father. "I took me only two minutes to take notes on two whole pages. It looks like I may not have to spend the rest of my life in Middle School after all!"

"Good thing," said Mr. Studious with a chuckle, "because your mother and I don't intend to support you forever."

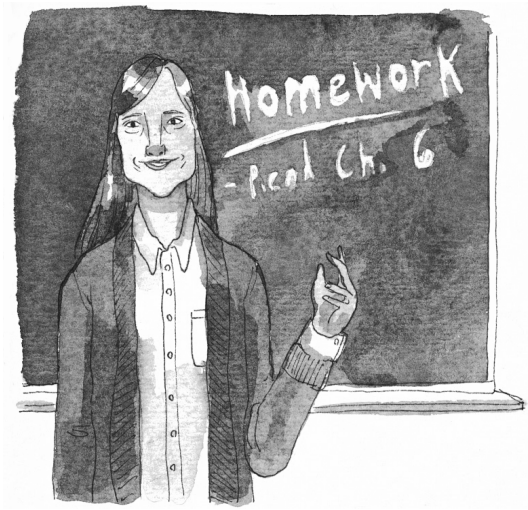
10. With this note taking technology, how much time could Germaine save? Now, what is the **total** time required to study for American history?

11. Were the seven hours Germaine planned to study enough for the work he needs to complete?

Project Idea: With a partner, find your reading rates for 5 pages of a novel, a history or science textbook, and calculate your rates for solving word problems. Use these rates to determine how much time it will take you to finish a novel, read a chapter in a textbook or solve 20 word problems.

Practice Problems

According to https://wmich.edu/asc/files/IncreasingYourReadingRate.pdf reading is something that takes a lot of our time – between reading for classes, reading for fun and all the extra reading assignments for research projects and papers, there may be limited time for other extra-curricular activities. Increasing the speed of reading will help cut back on some of this time!



Different types of material read at different rates. The average reader reads a:

Newspaper at 400-600 wpm

Literature/Stories at 300-400 wpm

Textbooks at 240-300 wpm

To establish a base reading rate, use the following process. This can be used for comparison to new techniques only.

- Pick out a book.
- Read for 3 minutes marking both the beginning and an ending place.
- Multiply # lines read by Avg. # words per line then divide by 3 to get the reading rate.

1. What does wpm represent in the statements above?

2. Due to space restrictions, a newspaper reporter is limited to 5000 words for a local news story. How long will it take the average reader to read that article?

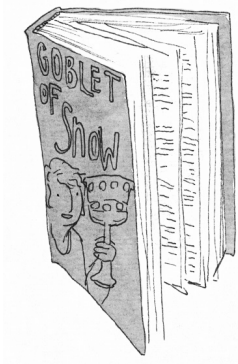
3. Mrs. Herstory, the social studies teacher, has assigned the class the reading of Chapter 6 in their textbook. The chapter consists of 18 pages with approximately 450 words on each page.

   a. How long will it take the average reader to read Chapter 6?

   b. Mrs. Herstory usually assigns not more than 30 minutes of reading homework per night. Does this assignment stay within her limit? Explain your thinking.

4. The English Department at Happy Valley School has adopted a new literature series. The 7th graders have been assigned "The Devil's Arithmetic." This novel contains 38,286 words. How long will it take the average reader to read the novel suggested? Explain your answer.

5. English classes at Happy Valley meet 5 days a week for 50 minutes per day. Of these 5 days, English teachers allow 20 minutes of reading time on Mondays, Wednesdays, and Fridays. Based on your answer to number 4 above, and if students only read the novel during English class, how many weeks should teachers plan for the unit on "The Devil's Arithmetic." Explain your thinking.



6. Use the process described above to establish your base reading rate for a new novel. Give your reading rate and explain how you calculated the *average number of words per line*.

7. In the process described above in the last bullet, why did you divide by 3 to get the reading rate?

8. Write a formula for calculating a reading rate if m represents the number of minutes read, Let l represents the number of lines read, and w represents the average number of words per line.

9. Debbie noticed that she had read 150 lines in 3 minutes. What would Debbie's unit rate (lines per minute) be? How many seconds per line would this be?

Thinking Through a Lesson Protocol

UNIT: Rates

Suggested time: Two to three class periods

Materials needed: A novel or history or science book to read to calculate reading rate

Standards

6.RP.A.2: Understand the concept of a unit rate a/b associated with a ratio a:b with b not equal to 0 and use rate language in the context of a ratio relationship.

6.RP.A.3: Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations.

6.RP.A.3B: Solve unit rate problems including those involving unit pricing and constant speed. For example, if it took 7 hours to mow 4 lawns, then at that rate, how many lawns could be mowed in 35 hours? At what rate were lawns being mowed?

Mathematical Practices:

MP1: Make sense of problems and persevere in solving them.

MP2: Reason abstractly and quantitatively.

MP6: Attend to precision.

MP7: Look for and make use of structure.

Setting up the Problem - Launch	
Selecting tasks and goal setting	(15 minutes) Ask students if they know their reading rate. Discuss why the rate may be different for different types of reading. Provide students with a common reading, one resource is suggested below. Time all students reading for 2 minutes and have them mark where they were when time was called. Have students count the number of lines they read, calculate the average number of words per line as a class, and use this to calculate their reading rate. Begin by multiplying the number of lines they read, by the average number of words per line (as determined by class), and then divide by 2. **Unlocking Children's Math Potential: 5 Research Results to Transform Math Learning** by Jo Boaler, Professor of Mathematics Education, Stanford University https://bhi61nm2cr3mkdgk1dtaov18-wpengine.netdna-ssl.com/wp-content/uploads/2017/03/teacher-article-youcubed2.pdf

Monitoring student work - Explore		
Part I – All Students		
Strategies and misconceptions - Anticipating	**Who - Selecting and sequencing**	**Questions and Statements - Monitoring**
(10 minutes) Have students read text and answer questions #1-2 individually. Share results in whole group discussion.		Are there other rates that we could find other than words per minute? Is this a unit rate? How do you know? Is there another way to write a different unit rate?
(15-20 minutes) Continue to read text and answer questions #3-6 with a partner. Have the whole group share out their answers for 3-6.		Is there a unit rate? What might it be? What makes it a unit rate?
(20-30 minutes) Continue reading and with a partner answer 7-11. Share whole group.		Is there a unit rate? What might it be? What makes it a unit rate?

Monitoring individual student work - Explore		
Part II – Specific Groups of Students		
Strategies and misconceptions - Anticipating	**Who - Selecting and sequencing**	**Questions and Statements - Monitoring**
For off-task students or for students that seem to be self-conscious about you listening to them share.		I am just listening or looking to find out how you are working on the problem. This helps me think about what we will do later.
For students that appear to be stuck. Also for when you are having a difficult time understanding their strategies.		Can you tell me a little about your reading? How would you describe the problem in your own words? What facts do you have? Could you try it with simpler numbers?
For students that want to ask you questions, these are ways to uncover their thinking and judge to what extent you want to respond.		Tell me what you've thought about so far. What do you know? Why are you interested in more information about that? Let me say a little about that part.

Managing the discussion – Summarize	
Parts of discussion - Connecting	**Questions and statements - Connecting**
Launching the discussion: Select the problems in questions #9-11 that students are struggling with or you wish to share out.	Will team1 start us off by sharing one way of working on this problem? Please raise your hand when you are ready to share your solution. What did you do first when you were working on this problem? Let's start by clearing up a few things about the problem. Let's list some key parts in this problem. What was unclear in the problem?
Eliciting and uncovering student strategies	Joe would you be willing to start us off? What have you found so far? Can you repeat that? Can you explain how you got that answer? How do you know? Walk us through your steps. Where did you begin? Can you show us?
Focusing on Mathematical Ideas	Can you explain why this is true? Does this method always work? How is Bob's method similar to Kelly's method? What do all the solutions have in common? What would happen if I changed the numbers to _____?

Managing the discussion – Summarize	
Parts of discussion - Connecting	**Questions and statements - Connecting**
Encouraging Interactions	Do you agree or disagree with Kahlil's idea? What do others think? Would someone be willing to repeat what Tom just said? Would anyone be willing to add on to what Sue just said?
Concluding the Discussion	Can anyone tell me some of the big idea that we learned today? How would you explain what we learned today to a 5^{th} grader? Some of the key points from our discussion today are . . . Tomorrow we will continue our exploration of _____ beginning with the idea from today that _____.
Post Lesson Notes	You may wish to assign the practice problems that you feel would benefit the students.

Solutions to Text Questions

1. What is Germaine's reading rate when he focuses?

   When Germaine focuses his reading rate is 8 minutes for 5 pages or 1.6 minutes per page or .675 (⅝) pages per minute.

2. How long will it take him to read the remaining 145 pages with the faster reading rate?

   145 pages × 1.6 minutes/page = 232 minutes

3. How long will it take Germaine to solve the remaining 18 problems?

   Using 7 minutes per problem, it will take Germaine 126 minutes or 2 hours and 6 minutes to solve the remaining 18 problems. (18 problems × 7 minutes/problem)

4. How long will it take Germaine to solve the remaining 18 problems if he used the second problem to estimate his rate of solving word problems?

   Using 6 minutes per problem, it will take Germaine 108 minutes of 1 hour 48 minutes to solve the remaining 18 problems.

5. How long will it take Germaine to solve these five problems?

   If he uses his faster rate (6 min per problem), the more difficult problems will take him 12 minutes per problem or 60 minutes (1 hour) to solve the five problems.

6. How much time should Germaine plan for completing the math homework?

   There are 18 problems total of which five are difficult. If Germaine uses his faster rate (6 min per problem), it will take him (13 x 6 = 78) for the easier problems and (5 x 12 = 60) for the more difficult problems for a total of (78 + 60= 138) or 2 hours and 18 minutes for the entire problem set.

7. Why do you think the reading rate he found earlier should not be used in planning his American History studying?

   Germaine's early reading rate was determined from reading a novel. Reading for information from an American History book would require more comprehension. A different rate should be used.

8. How long will it take Germaine to read the remaining 34 pages?

   If Germaine reads 2 pages of American History in 6 minutes, his reading rate is 3 minutes per page or 1 page for every 3 minutes. It will take Germaine 34 x 3 or 102 minutes, 1 hour 42 minutes, to complete reading 34 pages of American History.

9. How much time does Germaine need to add to his total for taking notes? Now what is the **total** time required to study for American history?

   *Germaine's note-taking at a rate of 2 pages in 5 minutes will result in a unit rate of 1 page per 2.5 minutes. For 36 pages the notetaking adds 90 minutes or 1 hour 30 minutes to the American History studying. The **total** time for American History will then be*

   (reading + note-taking) = total time or

   (1 hour 42 minutes + 1 hour 30 minutes) = 3 hours 12 minutes

10. With this note taking technology, how much time could he save? Now what is the **total** time required to study for American history?

    The note-taking technology allows Germaine to take notes at a rate of 2 pages for every 2 minutes. The unit rate would be 1 page per minute so for 34 pages, note-taking should take him 34 minutes. Originally it was going to take him 90 minutes so he is saving 56 minutes!

    Total time would then be:

    (reading + note-taking) = total time or

    (102 minutes + 34 minutes) = 136 minutes or 2 hour 16 minutes

11. Were the seven hours Germaine planned to study enough for the work he needs to complete?

    Germaine needs time to study for the three different content areas: the novel, mathematics, and American History. The rates and timing of each are as follows:

    Novel: remaining 145 pages at a rate of 8 min/5 pages = 232 minutes

    Math: 6 min/problem for 13 problems + 12 min/×problem for 5 problems

    = 78 + 60 = 138 minutes

    American History: 6min/2 pages for 34 pages + 34min/34 pages =

    = 3× 34 + 34 = 136 minutes.

    The TOTAL study time is 232 +138 +136 = 506

    8 hours 26 minutes. Germaine's original estimate was that he would need only 7 hours to study.

Solutions to Practice Problems

1. What does wpm represent in the statements above?

   "wpm" means words per minutes

2. Due to space restrictions, a newspaper reporter is limited to 5000 words for a local news story. How long will it take the average reader to read that article?

   If the average reader is reading at 400 wpm it will take 5000 words divided by 400 wpm or 12.5 minutes (12 minutes and 30 seconds) to read the article.

   If the average reader is reading at 600 wpm it will take 5000 words divided by 600 wpm or 8.3333333 minutes (8 minutes and 20 seconds) to read the article.

3. Mrs. Herstory, the social studies teacher, has assigned the class the reading of Chapter 6 in their textbook. The chapter consists of 18 pages with approximately 450 words on each page.

   a. How long will it take the average reader to read Chapter 6?

      First to find the number of words multiply 18 pages by 450 words per page to get 8100 words. At 240 wpm, it will take 8100 words/240 wpm or 33.75 minutes (33 minutes and 45 seconds). At 300 wpm it will take 8100 words/300 wpm or 27 minutes. The average reader will take between 27 minutes and 33.75 minutes.

   b. Mrs. Herstory usually assigns not more than 30 minutes of reading homework per night. Does this assignment stay within her limit?

      Yes if the student reads at 300 wpm; No if the student reads at 240 wpm. Explain your thinking. This assignment falls within the range of average readers, so some will be able to read in 30 minutes or less, others will take more than 30 minutes.

4. The English Department at Happy Valley School has adopted a new literature series. The 7th graders have been assigned "The Devil's Arithmetic." This novel contains 38,286 words. How long will it take the average reader to read the novel suggested? Explain your answer.

   If the average reader rate is 300 wpm and the novel is 38,286 words, it will take 38,286 words divided by 300 wpm or approximately 128 minutes (2 hours and 8 minutes).

   If the average reader rate is 400 wpm and the novel is 38,286 words, it will take 38,286 words divided by 400 wpm or approximately 98 minutes (1 hour 38 minutes).

5. English classes at Happy Valley meet 5 days a week for 50 minutes per day. Of these 5 days, English teachers allow 20 minutes of reading time on Mondays, Wednesdays, and Fridays. Based on your answer to number 4 above, and if students only read the novel during English class, how many weeks should teachers plan for the unit on "The Devil's Arithmetic." Explain your thinking.

   20 minutes of reading time on Mondays, Wednesdays, and Fridays is the equivalent of 60 minutes per week or 1 hour per week. In general terms the teachers should allow at least 2 weeks to accommodate most students. Average readers will need between 1.5 and 2 hours to read the novel.

6. Use the process described above to establish your base reading rate for a new novel. Give your reading rate and explain how you calculated the average number of words per line.

   Reading rates will vary for students. How students calculate the average number of words per line will also vary. Some students may just pick one line and use it as an average, others may use 2 or 3 lines to calculate an average. Lines may be consecutive or randomly selected.

7. In the process described above in the last bullet, why did you divide by 3 to get the reading rate?

   We divide by 3 to get the rate per minute. We read for 3 minutes.

8. Write a formula for calculating a reading rate if *m* represents the number of minutes read, *l* represents the number of lines read, and *w* represents the average number of words per line.

   Reading rate = lw/m

9. Debbie noticed that she had read 150 lines in 3 minutes. What would Debbie's unit rate (lines per minute) be?

   Debbie's reading rate is (150 lines/3 minutes) 50 lines per minute.

How many seconds per line would this be?

50 lines in 60 seconds is the same as 1 line in 60/50 seconds or 1.2 seconds per line.